STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 86
(For candidates admitted from the academic year 2023 – 2024 and thereafter)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE	:	**MAJOR CORE**
PAPER	:	**ORDINARY DIFFERENTIAL EQUATIONS**
SUBJECT CODE	:	**23MT/PC/OD14**
TIME	:	**3 HOURS** **MAX. MARKS: 100**

Q. No.	**SECTION A** $(5 \times 2 = 10)$ **Answer ALL questions**	CO	KL
1.	Define Wronskian and state wronskian condition for two linearly independent functions $x_1(t),\ x_2(t)$.	1	1
2.	State a non-homogenous linear system of $n -$ equations.	1	1
3.	Define an analytic function in power series expansion.	1	1
4.	State Gronwall Inequality.	1	1
5.	State Sturm-Liouville boundary-value problem.	1	1

Q. No.	**SECTION B** $(10 \times 1 = 10)$ **Answer ALL questions**	CO	KL
6.	The Wronskian of the functions $\{e^x,\ sinx,\ cosx\}$ is _______ a) 0 b) $2e^x$ c) $2sinx$ d) 1	2	2
7.	The particular solution of $t^2x'' - 2x = 2t - 1, 0 < t < \infty$ is _________ a) $\frac{1}{2} - t$ b) $1 - t$ c) $logt$ d) 0	2	2
8.	A solution matrix $\emptyset$ of the differential equation $X' = A(t)X, t \in I$ is a fundamental matrix $x' = A(t)x, t \in I$ if and only if a) $\det\emptyset \neq tr\ A$ b) $\det\emptyset = 0$ c) $\det\emptyset \neq 0$ d) None of these	2	2
9.	The general non-zero solution $e^{At+A\omega}$ of $x' = Ax$ is said to be a periodic function with period ω if the solution equals to a) e^c b) e^{At} c) $e^{A\omega}$ d) $log\omega$	2	2
10.	The Hermite equation $x'' - 2tx' + 2x = 0$ has an ordinary point at a) $t = 1$ b) $t = 0$ c) $t = \infty$ d) None of these	2	2
11.	For the Bessel's equation $t(t-1)^2(t+3)x'' + t^2x' - (t^2 + t - 1)x = 0, t \neq \{0, 1, -3\}$, the regular singular points are a) $0, 1, -3$ b) $1, -3$ c) $0, -3$ d) None of these	2	2

12.	If the linear forms $V_1,\ V_2$ satisfy the relation $V_1 = KV_2$ where K is a scalar, then $V_1,\ V_2$ are said to be ____________ a) Linearly dependent c) Linearly Independent b) Orthogonal d) None of these	2	2
13.	The linear combination of set of solutions is also a solution of ODE by the method called _________________ a) Principle of variation method c) Variable separable b) Principle of superposition d) Dominance Property	2	2
14.	A linear BVP $L(x) = a(t)x'' + b(t)x' + c(t)x = 0, A \leq t \leq B$ is singular if and only if ____________ a) Either $A = -\infty$ or $B = \infty$ only b) Both $A = -\infty$ and $B = \infty$ only c) $a(t) = 0$ for atleast one point t in $[A, B]$ only d) All of the above	2	2
15.	The eigen functions $\emptyset_m$ and $\emptyset_n$ are orthogonal with respect to the weight function $r(x)$ on $[a, b]$ if it satisfies the condition a) $\int_a^b \emptyset_m(x)\emptyset_n(x)\, r(x)dx = 0$ b) $\int_a^b \emptyset_m(x)\emptyset_n(x)\, r(x)dx = \infty$ c) $\int_a^b \emptyset_m(x)\emptyset_n(x)\, r(x)dx \neq 0$ d) $\int_a^b \emptyset_{mn}(x)\, dx \neq 0$	2	2

Q. No.	**SECTION C** $(\mathbf{2 \times 15 = 30})$ **Answer ANY TWO questions**	**CO**	**KL**
16.	a) State and prove Abel's formula for n^{th} order linear non-homogeneous differential equation. b) Solve the IVP $x'' - 2x' + x - 0; x(0) = 0, x'(0) = 1$ (10+5)	3	3
17.	a) Let$A; I \rightarrow M_n(R)$ be continuous. Suppose a matrix $\emptyset$ satisfies $X' = A(t)X, t \in I.$ Prove that $det\emptyset$ satisfies the equation $(det\emptyset)' = (trA)(det\emptyset)$. b) Compute ordinary and singular points of the ODE $(1 - x^2)y'' - 6xy' - 4y = 0$. State the condition under which the singular point becomes regular. (10+5)	3	3
18.	Find the solution to the ODE $y' = y - 1; y(0) = 2$ up to fourth approximation by Picard's approximation theorem.	3	3
19.	Consider a one-dimensional rod of length c whose end points are insulated and $f(x), 0 < x < c$ is the temperature distribution at $t = 0$. Formulate the boundary-value problem to this physical phenomena and determine its temperature function.	3	3

Q. No.	**SECTION D** $(2 \times 15 = 30)$ **Answer ANY TWO questions**	CO	KL
20.	a) Let $b_1, b_2, \dots . b_n$ be real and continuous functions defined on an interval I and $\emptyset_1, \emptyset_2, \dots . \emptyset_n$ are n solutions of $L(x)(t) = 0, t \in I$ existing on I. Prove that these solutions are linearly independent on I if and only if $W(t) \neq 0, t \in I$. b) State and prove orthogonal property of Legendre polynomials. (9+6)	4	4
21.	a) Prove that the set of all solutions of the system $x' = A(t)x$ on I forms an $n-$ dimensional vector space over the field of complex numbers. b) Prove Rodrigue's Formula $P_n(t) = \frac{1}{2^n (n)!} \frac{d^n}{dt^n} (t^2 - 1)^n$. (9+6)	4	4
22.	State and prove Picard's Existence theorem.	4	4
23.	Find the eigen value and eigen function of the ODE $\frac{d}{dx}\left[x \frac{dy}{dx}\right] + \frac{\alpha}{x} y = 0$ if $y'(1) = 0, y'(e^{2\pi}) = 0$.	4	4

Q. No.	**SECTION E** $(2 \times 10 = 20)$ **Answer ANY TWO questions**	CO	KL
24.	Suppose x_p is any particular solution of a non- homogeneous linear equation $L(x)(t) = x''(t) + b_1(t)x'(t) + b_2(t)x(t) = d(t),\ t \in I$ and that x_h is the general solution of the homogeneous equation $L(x) = 0$. Prove that $x = x_p + x_h$ is its general solution and also for n^{th} order non-homogeneous ODE.	5	5
25.	Determine characteristic values and $exp\ (tA)$ for the system $x' = Ax$, where $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 3 \\ 0 & 1 & 0 \end{bmatrix}$.	5	5
26.	Prove that the integral representation of Bessel's function is $\frac{1}{\pi} \int_0^{\pi} \cos(n\theta - t sin\theta)\, d\theta$.	5	5
27.	State Lipschitz continuous condition and check whether the IVP $x' = \frac{2}{t} x, x(0) = 0$ has unique solution.	5	5

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