STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 86
(For candidates admitted from the academic year 2023 – 2024)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE	**:**	**CORE**	
PAPER	**:**	**MATHEMATICAL STATISTICS**	
SUBJECT CODE	**:**	**23MT/PC/MS34**	
TIME	**:**	**3 HOURS**	**MAX. MARKS: 100**

Q. No.	**SECTION A** $(\mathbf{5 \times 2 = 10})$ **Answer ALL questions**	**CO**	**KL**
1.	State any two properties of characteristic function.	1	1
2.	Find the characteristic function of zero-one distribution.	1	1
3.	Define stochastic convergence.	1	1
4.	Define the χ^2 statistic.	1	1
5.	Define consistent estimator.	1	1

Q. No.	**SECTION B** $(\mathbf{10 \times 1 = 10})$ **Answer ALL questions**	**CO**	**KL**
6.	Let X be a random variable with characteristic function $\varphi_X(t)$. Then the characteristic function of the random variable $Y = X + b$ is ____________ (i) $e^{it}\varphi_X(bt)$ (ii) $e^{bit}\varphi_X(t)$ (iii) $e^{ibt}\varphi_X(bt)$	2	2
7.	If X and Y are two independent random variable with characteristic function $\varphi_X(t)$ and $\varphi_Y(t)$ respectively. Then the characteristic function of $X + Y$ is ____________ (i) $\varphi_X(t) + \varphi_Y(t)$ (ii) $\varphi_X(t)\varphi_Y(t)$ (iii) $\varphi_X(t) - \varphi_Y(t)$	2	2
8.	When n is an integer, $\Gamma(n) =$___________ (i) $(n-1)!$ (ii) $n!$ (iii) $(n+1)!$	2	2
9.	The density function of Cauchy distribution is ____________ (i) $\frac{\lambda}{\pi(\lambda^2+(x-\mu)^2)}$ (ii) $\frac{\lambda}{\pi(\lambda^2-(x-\mu)^2)}$ (iii) $\frac{\lambda}{\pi(\lambda^2-(x+\mu)^2)}$	2	2
10.	The Bernoulli law of large numbers is a special case of ____________________ law of large numbers (i) Chebyshev's (ii) Poisson's (iii) Khintchin's	2	2

11.	The random variable X_n has an asymptotically normal distribution $N(np;\ \sqrt{npq})$ by replacing y_1 and y_2 with ____________ (ii) $y_1 - \frac{1}{2\sqrt{npq}};\ y_2 + \frac{1}{2\sqrt{npq}}$ (ii) $y_1 - \frac{1}{2\sqrt{npq}};\ y_2 - \frac{1}{2\sqrt{npq}}$ (iii) $y_1 + \frac{1}{2\sqrt{npq}};\ y_2 - \frac{1}{2\sqrt{npq}}$	2	2
12.	A random sample is called simple if the random variables $X_1, \dots X_n$ are ____________ (i) independent (ii) dependent (iii) both	2	2
13.	If the statistic $\bar{X}$ and S are independent, the random variables X_k have the ___________________ distribution. (i) Binomial (ii) Normal (iii) Poisson	2	2
14.	If T is an unbiased estimator for θ, then T^2 is a biased estimator for ________________ (i) θ^2 (ii) θ (iii) θ^3	2	2
15.	Characteristics of a good estimator are (i) consistency (ii) biasedness (iii) both	2	2

Q. No.	**SECTION C ($2 \times 15 = 30$)** **Answer ANY TWO questions**	**CO**	**KL**
16.	If $F(x)$ and $\varphi(t)$ denote respectively the distribution function and the characteristic function of the random variable X, and $a+h$, $a-h$ $(h > 0)$ are continuity points of the distribution function $F(x)$, then prove that $F(a+h) - F(a-h) = \lim_{T\to\infty} \frac{1}{\pi}\int_{-T}^{T} \frac{\sin ht}{t}\ e^{-iat}\varphi(t)\ dt.$	3	3
17.	a) State and prove De-Moivre Laplace theorem. b) Derive the raw moments m_k for the beta distribution. (10+5)	3	3
18.	Find the distribution of the random variable $(\bar{X}, S)$ and hence deduce that the random variable $Z = nS^2$ has a χ^2 distribution with $n-1$ degrees of freedom.	3	3
19.	State and Prove Cramer-Rao inequality.	3	3

Q. No.	**SECTION D ($2 \times 15 = 30$)** **Answer ANY TWO questions**	**CO**	**KL**
20.	Prove that the characteristic function of two dependent random variables may be equal to the product of their characteristic functions by using a suitable example.	4	4

21.	a) Prove that the variance of a random variable X equals zero if and only if X has a one point distribution b) Find the distribution of the sample mean $\bar{X}$ where X has a normal distribution. (7+8)	4	4
22.	State Levy-Cramer theorem and prove that if the sequence $\{F_n(x)\}$ $(n = 1, 2, \dots)$ of distribution functions is convergent to the distribution function $F(x)$, then the corresponding sequence of characteristic functions $\{\varphi_n(t)\}$ converges at every point t $(-\infty < t < \infty)$ to the function $\varphi(t)$ which is the characteristic function of the limit distribution function $F(x)$ and the convergence to $\varphi(t)$is uniform with respect to t in every finite interval on the $t - axis$.	4	4
23.	Define the Student's t statistic and derive its density function.	4	4

Q. No.	**SECTION E** $(\mathbf{2 \times 10 = 20})$ **Answer ANY TWO questions**	CO	KL
24.	Find the characteristic function and moments for the random variable X which has a Poisson distribution and takes on values $x_k = k$, where k is any non-negative integer.	5	5
25.	Define the Gamma distribution and find its characteristic function, raw moments m_k and central moment μ_2.	5	5
26.	Prove that the sequence of random variables is stochastically convergent to zero if and only if the sequence of distribution functions of these random variables is convergent to the distribution function $F(x) = \begin{cases} 0 & for\ \ x \leq 0 \\ 1 & for\ \ x > 0 \end{cases}$ at every continuity point of $F(x)$.	5	5
27.	If T_1 and T_2 be two unbiased estimators of $\gamma(\theta)$ with variance σ_1^2, σ_2^2 and correlation ρ, what is the best unbiased linear combination of T_1 and T_2 and what is the variance of such a combination?	5	5

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