STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 86
(For candidates admitted from the academic year 2023 – 2024 and thereafter)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2024
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : MAJOR CORE
PAPER : GRAPH THEORY
SUBJECT CODE : 23MT/PC/GT14
TIME : 3 HOURS **MAX. MARKS: 100**

Q. No.	SECTION A ($5 \times 2 = 10$) Answer ALL questions	CO	KL
1.	Find incidence matrix for $K_{2,3}$.	1	1
2.	In a complete bipartite graph, what kind of relationship exists between a maximum matching and a minimum covering of a graph *G*?	1	1
3.	Is every critical graph a block?	1	1
4.	Define independent dominating set with an example.	1	1
5.	Draw Q_4.	1	1

Q. No.	SECTION B ($10 \times 1 = 10$) Answer ALL questions	CO	KL
6.	Let *G* be a graph with $\nu - 1$ edges. Then *G* is _____ (a) not connected (b) cycle (c) tree (d) all the above	2	2
7.	A simple connected graph that has exactly two vertices which are not cut vertices is a _____. (a) path (b) cycle (c) complete graph (d) all the above	2	2
8.	Every *k*-regular bipartite graph is _____ (a) not factorable (b) 1-factorable (c) 2-factorable (d) none of these	2	2
9.	A graph is _____if $\alpha(G - e) > \alpha(G)$ for all $e \in E$ (a) β-critical (b) $\alpha\beta$-critical (c) $\alpha + \beta$-critical (d) α-critical	2	2
10.	If *G* is a tree then $\pi_k(G)$ = _____ (a) $k(k-1)^{\nu-1}$ (b) $k(k+1)^{\nu-1}$ (c) $k(k-1)^{\nu-2}$ (d) $(k-1)(k)^{\nu-1}$	2	2
11.	The edge-chromatic number of the Petersen graph *G*(10, 15) is _____ (a) 5 (b) 2 (c) 3 (d) 4	2	2

12.	The minimal domination number of path on 5 vertices is _____ (a) 1 (b) 2 (c) 3 (d) 4	2	2
13.	If G is a simple planar graph with $v \geq 3$ then (a) $\epsilon \geq 3v - 5$ (b) $\epsilon \geq 3v - 6$ (c) $\epsilon \leq 3v - 6$ (d) $\epsilon \leq 3v - 5$	2	2
14.	The diameter of a complete graph K_n is _____ (a) 0 (b) 1 (c) n (d) $n - 1$	2	2
15.	Which of the following graph is not vertex-transitive graph? (a) Complete graph (b) hypercube network (c) De Bruijn network (d) Circulant network	2	2

Q. No.	**SECTION C ($2 \times 15 = 30$)** **Answer ANY TWO questions**	CO	KL
16.	(i) Show that every nontrivial loopless connected graph has at least two vertices that are not cut vertices. (ii) Prove that an edge e of G is a cut edge of G if and only if e is contained in no cycle of G. **(7 + 8)**	3	3
17.	(i) Prove that $\kappa \leq \kappa' \leq \delta$. (ii) State and prove Berge's theorem. **(7 + 8)**	3	3
18.	(i) Prove that if G is bipartite with $p \geq \Delta$ then there exist p disjoint matchings $M_1, M_2, \ldots, M_P$ of G such that $E = M_1 \cup M_2 \cup \ldots \cup M_p$ and for $1 \leq i \leq p, \lceil \epsilon/p \rceil \leq \lvert M_i \rvert \leq \{\epsilon/p\}$ with an example of timetable problem. (ii) Let G be a connected graph that is not an odd cycle. Then prove that G has a 2-edge coloring in which both colors are represented at each vertex of degree at least two. **(9 + 6)**	3	3
19.	(i) Show that K_5 is nonplanar. (ii) Explain the basic principles of Network design. **(7 + 8)**	3	3

Q. No.	**SECTION D ($2 \times 15 = 30$)** **Answer ANY TWO questions**	CO	KL
20.	Prove the following: (i) A graph is bipartite if and only if it contains no odd cycle. (ii) Every nontrivial tree has at least two vertices of degree one. **(8 + 7)**	4	4

21.	(i) Using Hall's theorem prove that in a bipartite graph, the number of edges in a maximum matching is equal to the number of vertices in a minimum covering. (ii) If $\delta > 0$, then prove that $\alpha' + \beta' = \nu$. **(7 + 8)**	4	4
22.	(i) Suppose G is k −critical graph with a 2-vertex cut $\{u, v\}$ then prove that $G = G_1 \cup G_2$ where G_i is a $\{u, v\}$ – component of type i. (ii) State and prove Brook's theorem. **(6 + 9)**	4	4
23.	(i) State and prove Euler's formula. (ii) Draw Circulant network $G(12, \{1, 2, 3\})$ and Kautz network $K(2, 3)$. **(8 + 7)**	4	4

Q. No.	**SECTION E ($2 \times 10 = 20$)** **Answer ANY TWO questions**	**CO**	**KL**
24.	Explain shortest path problem using Dijkstra's algorithm.	5	5
25.	If G is simple graph, then prove that $\pi_k(G) = \pi_k(G - e) - \pi_k(G.e)$ for any edge e of G. Hence obtain $\pi_k(K_{1,3})$.	5	5
26.	Prove that every planar graph is 5-vertex-colorable.	5	5
27.	Explain any three properties of hypercube network.	5	5