STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted during the academic year 2023–24)

B. Sc. DEGREE EXAMINATION, NOVEMBER 2024
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE   : **MAJOR – CORE**
PAPER   : **ELEMENTS OF GRAPH THEORY**
SUBJECT CODE : **23MT/MC/EG34**
TIME   : **3 HOURS**   **MAX. MARKS: 100**

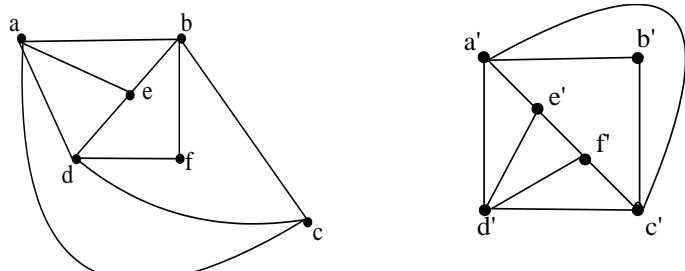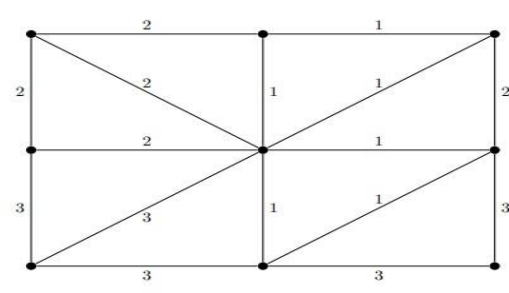| Q. No.	SECTION A ($5 \times 2 = 10$) **Answer ANY FIVE questions**	CO	KL
1.	Define complete graph and null graph	1	1
2.	Define isomorphism in graphs.	1	1
3.	What is a graphic sequence?	1	1
4.	Define eulerian graph.	1	1
5.	Define eccentricity of vertices and radius of a graph.	1	1
6.	Distinguish between a spanning path and a semipath of a graph.	1	1

Q. No.	SECTION B ($10 \times 1 = 10$) **Answer ALL questions**	CO	KL
7.	The adjacency matrix of a graph is _____. (a) Hermitian (b) asymmetric (c) symmetric (d) orthogonal	2	2
8.	An induced subgraph of a _____ graph is complete. (a) regular (b) isomorphic (c) complementary (d) complete	2	2
9.	Any two _____ graphs determine the same partition. (a) isomorphic (b) complete (c) regular (d) eulerian	2	2
10.	Every non-trivial connected graphs has at least _____ points which are not cut points. (a) three (b) two (c) one (d) five	2	2
11.	An eulerian graph G is arbitrary traversable from a vertex v in G if and only if every _____ in G contains v. (a) path (b) trail (c) cycle (d) walk	2	2
12.	Petersen graph is _____. (a) hamiltonian (b) non-hamiltonian (c) eulerian (d) complete	2	2
13.	Every tree is a _____ graph. (a) hamiltonian (b) complete (c) bipartite (d) eulerian	2	2

14.	Any connected (p, q) graph with $p + 1 = q$ is a _____. (a) tree (b) cycle (c) walk (d) block	2	2
15.	All _____ of a connected graph are included in a minimum spanning tree of a graph. (a) edges (b) vertices (c) paths (d) cycles	2	2
16.	The sequential representation of a graph is by means of _____. (a) incidence matrix (b) adjacency matrix (c) linked lists (d) degree sequence	2	2

Q. No.	SECTION C ($2 \times 15 = 30$) **Answer ANY TWO questions**	CO	KL
17.	a) Prove that any self complementary graph has $4n$ or $4n+1$ points. b) If G_1 is a (p_1, q_1) graph and G_2 is a (p_2, q_2) graph, then prove that $G_1 \times G_2$ is a $(p_1 p_2, q_1 p_2 + q_2 p_1)$ graph. **(7+8)**	3	3
18.	a) Prove that a graph G with at least two points is bipartite if and only if all its cycles are of even length. b) Prove that if G is connected, then $\bar{G}$ is connected. **(12+3)**	3	3
19.	a) State and prove Dirac's Theorem on Hamiltonian Graphs. b) Explain Fleury's algorithm and give its significance. **(10+5)**	3	3
20.	Let G be a (p, q) graph. Then prove that the following statements are equivalent: (i) G is a tree. (ii) Every two points of G are joined by a unique path. (iii) G is connected and $p = q + 1$ (iv) G is acyclic and $p = q + 1$	3	3

Q. No.	SECTION D ($2 \times 15 = 30$) **Answer ANY TWO questions**	CO	KL
21.	Prove that the maximum number of lines among all p point graphs with no triangles is $\left[\frac{p^2}{4}\right]$.	4	4
22.	a) Prove that a graph G is connected if and only if for any partition of V into subsets V_1 and V_2 there is a line of G joining a point of V_1 to a point of V_2. b) Prove that if G is a block then any two points of G lie on a common cycle. **(7+8)**	4	4

23.	Show that the following statements are equivalent for a connected graph: i) G is Eulerian. ii) Every point of G has even degree. iii) The set of edges of G can be partitioned into cycles.	4	4
24.	a) State and prove Euler's formula. b) If the adjacency matrix of a graph is $A = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$, find its path matrix. **(8+7)**	4	4

Q. No.	SECTION E ($2 \times 10 = 20$) **Answer ANY TWO questions**	CO	KL
25.	a) Show that in any group of two or more people, there are always two with exactly the same number of friends inside the group. b) Check whether the following graphs are isomorphic: **(5+5)**	5	5
26.	a) Show that the partition $P = (6,6,5,4,3,3,1)$ is not graphic. b) Prove that in a graph G, any $u - v$ walk contains a $u - v$ path. **(5+5)**	5	5
27.	In any connected plane (p, q) graph $(p \geq 3)$ with r faces, prove that $q \geq 3r/2$ and $q \leq 3p - 6$ and hence prove that K_5 is not planar.	5	5
28.	Find a minimal spanning tree of the following graph using Kruskal's algorithm: 	5	5

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