STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 86
(For candidates admitted from the academic year 2023 – 2024)

B.Sc. DEGREE EXAMINATION, NOVEMBER 2024
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE	**:**	**MAJOR CORE**
PAPER	**:**	**DIFFERENTIAL EQUATIONS**
SUBJECT CODE	**:**	**23MT/MC/DE34**
TIME	**:**	**3 HOURS** **MAX. MARKS: 100**

Q. No.	**SECTION A** $(\mathbf{5 \times 2 = 10})$ **Answer ANY FIVE questions**	CO	KL
1.	Find the general solution of $\frac{d^2y}{dx^2} - 6\frac{dy}{dx} + 8y = 0$.	1	1
2.	Find the particular integral of $(D^2 - 4)y = e^{2x}$.	1	1
3.	How do we reduce linear equations with variable coefficients into a linear equations with constant coefficients.	1	1
4.	Find the complete solution of $p^2 + q^2 = m^2$.	1	1
5.	Write the auxiliary equation of the differential equation $\left(D^3 - 3D^2D' + 2DD'^2\right)Z = 0$.	1	1
6.	State Hooke's law.	1	1

Q. No.	**SECTION B** $(\mathbf{10 \times 1 = 10})$ **Answer ALL questions**	CO	KL
7.	The roots of the auxiliary equation $m^3 - 3m + 2 = 0$ is ___________________ (a) 1, -1, 2 (b) 1, -1, -2 (c) 1, 1, -2 (d) -1, -1, 2	2	2
8.	If $\phi(-\alpha^2) \neq 0$ then P. I. = $\frac{1}{\phi(D^2)} \sin \alpha x$ is ______________ (a) $\frac{1}{\phi(\alpha^2)} \sin \alpha x$ b) $\frac{1}{\phi(-\alpha^2)} \sin \alpha x$ (c) $\frac{1}{\phi(\alpha^{\square})} \sin \alpha x$ (d) $\frac{1}{\phi(-\alpha^{\square})} \sin \alpha x$	2	2
9.	By the method of variation of parameters a particular solution of a non-homogenous equation can be found when ___________________ (a) two linearly independent solutions of the homogenous equation are unknown (b) two linearly dependent solutions of the homogenous equation are unknown (c) two linearly independent solutions of the homogenous equation are known (d) two linearly dependent solutions of the homogenous equation are known	2	2

10.	In the differential equation $P\frac{\partial z}{\partial x}+Q\frac{\partial z}{\partial y}+R=0$ of first order and first degree ________________ (a) z is independent and x and y are dependent variables. (b) z is dependent and x and y are independent variables. (c) x is independent and z and y are dependent variables. (d) y is independent and x and z are dependent variables.		
11.	$\frac{\partial^2 y}{\partial t^2}=a^2\frac{\partial^2 y}{\partial x^2}$ is a partial differential equation of order ___________ (a) one (b) two (c) three (d) zero	2	2
12.	The equation $Pp+Qq=R$ is called a quasi linear equation. When P, Q, R are independent of Z then it is known as __________ (a) linear equation (b) quadratic equation (c) both (a) and (b) (d) polynomial equation	2	2
13.	For $f(D,D')z=F(x,y)$, if $F(x,y)=e^{ax+by}$ then P. I. = ___________ (a) $\frac{e^{ax+by}}{f(-a,-b)}$ (b) $\frac{e^{ax+by}}{f(-a^2,-b^2)}$ (c) $\frac{e^{ax+by}}{f(a^2,b^2)}$ (d) $\frac{e^{ax+by}}{f(a,b)}$	2	2
14.	If the auxiliary equation of $r+6s+9t=0$ is $(m+3)^2=0$ then the complete integral is ________________ (a) $z=\phi_1(y-3x)+x\phi_2(y-3x)$ (b) $z=\phi_1(y+3x)+x\phi_2(y+3x)$ (c) $z=\phi_1(y-3x)+\phi_2(y+3x)$ (d) $z=\phi_1(y-3x)+x\phi_2(y+3x)$	2	2
15.	The algebraic sum of the voltage drops in a simple closed electric circuit is ____________________ (a) 1 (b) ∞ (c) $-\infty$ (d) 0	2	2
16.	The equation $F=ma$ where F is a force, m is a mass and a is a acceleration is _____________ (a) Newton's first law (b) Newton's second law (c) Newton's third law (d) Hooke's law	2	2

Q. No.	**SECTION C $(2 \times 15 = 30)$** **Answer ANY TWO questions**	CO	KL
17.	An RCL circuit connected in series has a resistance of 5 ohms, an inductance of 0.05 henry, a capacitor of 4×10^{-4} farad and an applied alternating emf of 200 cos $100t$ volts. Find an expression for the current flowing through this circuit if the initial current and the initial charge on the capacitor are both zero.	3	3
18.	(i) Solve the equations: $\frac{dx}{dt} + y = \sin t + 1$; $\frac{dy}{dt} + x = \cos t$ (ii) Solve: $x^2 \frac{d^2y}{dx^2} + 7x \frac{dy}{dx} + 13y = \log x$. **(9 + 6)**	3	3
19.	(i) Find the differential equation of all planes which are at constant distance k from the origin. (ii) Solve $\frac{\partial z}{\partial x} = 6x + 3y, \frac{\partial z}{\partial y} = 3x - 4y$. **(8 + 7)**	3	3
20.	Solve $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} + \frac{\partial^2 z}{\partial y^2} = x^2 + xy + y^2$.	3	3

Q. No.	**SECTION D $(2 \times 15 = 30)$** **Answer ANY TWO questions**	CO	KL
21.	(i) Show that the solution of the differential equation $\frac{d^2y}{dt^2} + 4y = A \sin pt$ which is such that $y = 0$ and $\frac{dy}{dt} = 0$ when $t = 0$, is $y = A\frac{(\sin pt - \frac{1}{2}p \sin 2t)}{4-p^2}$ if $p \neq 2$. If $p = 2$, show that $y = \frac{A(\sin 2t - 2t \cos 2t)}{8}$. (ii) Solve $(D-1)^2 y = x$. **(9 + 6)**	4	4
22.	(i) By variation of parameters method, solve $(x-1)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = (x-1)^2$ given that x and e^x are the particular integrals of the equation without the right hand member. (ii) Solve $x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = x$. **(8 + 7)**	4	4
23.	(i) Obtain the complete solution of $p + q = \sin x + \sin y$. (ii) Using Lagrange's Method, solve $(z-y)p + (x-z)q = (y-x)$. **(7 + 8)**	4	4

24.	(i) Solve $r - 4s + 4t = e^{2x+y}$. (ii) A 10-kg mass is attached to a spring having a spring constant of 140 N/m. The mass is started in motion from the equilibrium position with an initial velocity of 1 m/sec in the upward direction and with an applied external force $F(t) = 5\sin t$. Find the subsequent motion of the mass if the force due to air resistance is $-90\dot{x}\ N$. **(7 + 8)**	4	4

Q. No.	SECTION E $(\mathbf{2 \times 10 = 20})$ Answer ANY TWO questions	CO	KL
25.	Solve $(D^2 - 4D + 3)y = e^{-x}\sin x$.	5	5
26.	Solve: $(5+2x)^2\frac{d^2y}{dx^2} - 6(5+2x)\frac{dy}{dx} + 8y = 6x$.	5	5
27.	Form the partial differential equation by eliminating f from $z = xy + f(x^2 + y^2 + z^2)$.	5	5
28.	Explain spring problem and hence obtain the motion of a vibrating spring.	5	5