

**STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086**  
**(For candidates admitted during the academic year 2023–24 & thereafter)**

**B. Sc. DEGREE EXAMINATION, NOVEMBER 2024**  
**BRANCH I - MATHEMATICS**  
**FIRST SEMESTER**

**COURSE : MAJOR – CORE**  
**PAPER : DIFFERENTIAL CALCULUS**  
**SUBJECT CODE : 23MT/MC/DC14**  
**TIME : 3 HOURS**

**MAX. MARKS : 100**

Q. No.	SECTION A (5 × 2 = 10)	CO	KL
	<b>Answer ANY FIVE questions</b>		
1.	State Leibnitz's theorem.	1	1
2.	Find $y_2$ if $y = x^2 \log x$ .	1	1
3.	Define chord of curvature.	1	1
4.	Write the equation of the envelope of the family of straight lines.	1	1
5.	When is a function $f(x, y)$ said to have a maximum value at a point $(a, b)$ of the domain of $f(x, y)$ ?	1	1
6.	Write the property of spiral of Archimedes.	1	1

Q. No.	SECTION B (10 × 1 = 10)	CO	KL
	<b>Answer ALL questions</b>		
7.	If $y = e^{ax}$ then $D^n(e^{ax})$ is _____ (a) $ae^{ax}$ (b) $a^n e^{ax}$ (c) $(ae^{ax})^n$ (d) $(ae^{ax})^{\frac{1}{n}}$	2	2
8.	If $y = \tan^{-1} x$ then $y_1$ is _____. (a) $\frac{1}{1+x^2}$ (b) $\frac{1}{1-x^2}$ (c) $\frac{1}{x^2-1}$ (d) $\frac{1}{\sqrt{1-x^2}}$	2	2
9.	The radius of curvature for pedal equation $P = f(r)$ is _____ (a) $\rho + r \frac{dr}{dP}$ (b) $\rho - r \frac{dr}{dP}$ (c) $\rho - r \frac{dP}{dr}$ (d) $\rho + r \frac{dP}{dr}$	2	2
10.	The equation of the circle of curvature is _____ (a) $(x - \bar{x})^2 + (y - \bar{y})^2 = \rho^2$ (b) $(x - \bar{x})^2 - (y - \bar{y})^2 = \rho^2$ (c) $(x + \bar{x})^2 + (y + \bar{y})^2 = \rho^2$ (d) $(x + \bar{x})^2 - (y + \bar{y})^2 = \rho^2$	2	2
11.	The evolute of a curve is the envelope of its _____ (a) tangent      (b) straight line      (c) normal      (d) curve	2	2
12.	The envelope of the paths of projectile in vacuum from the same point with the same velocity in the same vertical plane is a _____ with the point of projection as focus. (a) ellipse      (b) hyperbola      (c) circle      (d) parabola	2	2

13.	If a function $f(x, y)$ is a maximum or minimum at $x = a, y = b$ and if the first partial derivatives $f_x(a, b)$ and $f_y(a, b)$ exist, then _____ (a) $f_x(a, b) = 0$ & $f_y(a, b) \neq 0$ (b) $f_x(a, b) \neq 0$ & $f_y(a, b) \neq 0$ (c) $f_x(a, b) \neq 0$ & $f_y(a, b) = 0$ (d) $f_x(a, b) = 0$ & $f_y(a, b) = 0$	2	2
14.	Suppose $u = f(x_1, x_2, \dots, x_n)$ is a function of $n$ variables which are connected by $m$ equations ( $m < n$ ) then $n - m$ variables are _____ (a) independent    (b) either independent or dependent (c) dependent    (d) neither independent nor dependent	2	2
15.	The evolute of the cardioid is _____ (a) limaçon    (b) cardioid (c) lemniscate    (d) rose lemniscate	2	2
16.	If the tangent at a double point $P$ on a plane curve be real and distinct, the double point is called a _____ (a) node    (b) cusp (c) isolated point    (d) double cusp	2	2

Q. No.	SECTION C ( $2 \times 15 = 30$ ) Answer ANY TWO questions	CO	KL
17.	(i) Determine $y_n$ if $y = e^x \sin^2 x$ . (ii) If $y = \frac{x^2+x-1}{x^3+x^2-6x}$ , find $y_n$ .    (7 + 8)	3	3
18.	(i) Find the radius of curvature of $y = xe^{-x}$ at its maximum point. (ii) Find the equation of the circle of curvature at the point (3, 1) on the curve $y = x^2 - 6x + 10$ .    (7 + 8)	3	3
19.	Find all the maxima and minima of the function $4x^2 - xy + 4y^2 + x^3y + xy^3 - 4$ .	3	3
20.	Classify double points on the curve and derive the conditions for the existence of double points on an algebraic curve.	3	3

Q. No.	SECTION D ( $2 \times 15 = 30$ ) Answer ANY TWO questions	CO	KL
21.	If $y = \cos(m \sin^{-1} x)$ , show that (i) $(1 - x^2)y_2 - xy_1 + m^2y = 0$ (ii) $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} + (m^2 - n^2)y_n = 0$ . Also find the value of $y_n$ when $x = 0$ .	4	4

22.	(i) Show that the chord of the curvature through the pole of the curve $r^m = a^m \cos m\theta$ is $\frac{2r}{m+1}$ . (ii) Find the centre of curvature of the curve $x = e^{-2t} \cos 2t, y = e^{-2t} \sin 2t$ at $t = 0$ . (7 + 8)	4	4
23.	(i) Show that the pedal equation of the envelope of the line $x \cos 2\alpha + y \sin 2\alpha = 2a \cos \alpha$ , where $\alpha$ is the parameter is $p^2 = \frac{4}{3}(r^2 - a^2)$ . (ii) Find the envelopes of the family of straight lines $y = mx + \sqrt{a^2m^2 + b^2}$ , where $m$ is the parameter. (7 + 8)	4	4
24.	Find the maximum value of $x^2y^3z^4$ subject to the condition $x + y + z = 18$ .	4	4

Q. No.	SECTION E (2 × 10 = 20) Answer ANY TWO questions	CO	KL
25.	If $x = a(\theta + \sin \theta), y = a(1 - \cos \theta)$ verify that $\frac{d^2y}{dx^2} = \frac{1}{4a} \sec^4 \frac{\theta}{2}$ .	5	5
26..	Determine the evolute of the curve whose equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ .	5	5
27.	If the centre of a circle lies upon the parabola $y^2 = 4ax$ and the circle passes through the vertex of the parabola. Show that the envelope of the circle is $y^2(2a + x) + x^3 = 0$ .	5	5
28.	Brief about the curve cycloid and mention its characteristic properties.	5	5

