STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 86
(For candidates admitted from the academic year 2023 – 2024 and thereafter)

B.Sc. DEGREE EXAMINATION, NOVEMBER 2024
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE	**:**	**MAJOR CORE**
PAPER	**:**	**ALGEBRA & TRIGONOMETRY**
SUBJECT CODE	**:**	**23MT/MC/AT13**
TIME	**:**	**3 HOURS**

MAX. MARKS: 100

Q. No.	SECTION A $(\mathbf{5 \times 2 = 10})$ Answer ANY FIVE questions	CO	KL
1.	Remove the fractional coefficients from the equation $x^3 - \frac{1}{4}x^2 + \frac{1}{3}x - 1 = 0$.	1	1
2.	Show that the coefficient of x^n in the infinite series $1 + \frac{b+ax}{1!} + \frac{(b+ax)^2}{2!} + \ldots + \frac{(b+ax)^n}{n!} + \ldots$ is $\frac{e^b a^n}{n!}$.	1	1
3.	Find the eigen values of the following matrix $\begin{bmatrix} 8 & -4 \\ 2 & 2 \end{bmatrix}$.	1	1
4.	Give the expansion of $\tan\theta$ in series of ascending powers of θ.	1	1
5.	Prove that $\cosh^2 x + \sinh^2 x = \cosh 2x$.	1	1
6.	Express $\sinh^{-1} x$ in terms of logarithmic functions.	1	1

Q. No.	SECTION B $(\mathbf{10 \times 1 = 10})$ Answer ALL questions	CO	KL
7.	The equation whose roots are the roots of $x^5 + 6x^4 + 6x^3 - 7x^2 + 2x - 1 = 0$ with signs changed is _________________. a) $x^5 - 6x^4 - 6x^3 - 7x^2 + 2x - 1 = 0$ b) $x^5 - 6x^4 + 6x^3 + 7x^2 + 2x + 1 = 0$ c) $x^5 - 6x^4 + 6x^3 - 7x^2 + 2x + 1 = 0$ d) $x^5 + 6x^4 - 6x^3 - 7x^2 + 2x - 1 = 0$	2	2
8.	For a biquadratic equation whose roots are in arithmetic progression, the general roots can be assumed to be ___________________. a) $\alpha - 3k, \alpha - k, \alpha + k, \alpha + 3k$ b) $\alpha - 2k, \alpha - k, \alpha + k, \alpha + 2k$ c) $\alpha - 6k, \alpha - k, \alpha + k, \alpha + 6k$ d) $\alpha - 2k, \alpha - k, \alpha + k, \alpha + 3k$	2	2

9.	$\frac{e - e^{-1}}{2} =$ _______________________. a) $1 + \frac{1}{2!} + \frac{1}{4!} + \cdots + \infty$ b) $\frac{1}{3!} + \frac{1}{5!} + \cdots + \infty$ c) $1 + \frac{1}{3!} + \frac{1}{5!} + \cdots + \infty$ d) $\frac{1}{2!} + \frac{1}{4!} + \cdots + \infty$	2	2
10.	$-\frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \cdots + \infty =$ ____________________ a) log 2 + 1 b) log 2 c) log 2 – 1 d) 1 – log 2	2	2
11.	The sum of the elements on the main diagonal of a matrix *A* is equal to the sum of its ________________. a) eigenvectors b) invariants c) eigenvalues d) elementary divisors	2	2
12.	An application of Cayley Hamilton theorem is to find the ________________ of the matrix. a) eigen vectors b) higher powers c) eigen values d) determinant	2	2
13.	The expansion of $sin^n\theta$ will be in terms of cosines of multiples of θ if *n* is an ______________ integer. a) odd b) even c) both odd and even d) none of the above	2	2
14.	$\tan n\,\theta$ is equivalent to ___________. a) $\frac{nC_1 \tan\theta - nC_3 \tan^3\theta + nC_5 \tan^5\theta \ldots}{1 - nC_2 \tan^2\theta + nC_4 \tan^4\theta \ldots}$ b) $nC_1 \tan\theta - nC_3 \tan^3\theta + nC_5 \tan^5\theta \ldots$ c) $1 - nC_2 \tan^2\theta + nC_4 \tan^4\theta \ldots$ d) $\frac{nC_1 \tan\theta + nC_3 \tan^3\theta + nC_5 \tan^5\theta \ldots}{1 + nC_2 \tan^2\theta + nC_4 \tan^4\theta \ldots}$	2	2

15.	$\cosh 2x =$ ________________. a) $2cosh^2x - 1$ b) $1 + 2sinh^2x$ c) $\frac{1+tanh^2x}{1- tanh^2x}$ d) All of the above	2	2
16.	If n is an integer, then $i^i =$ ____________________. $a)\ e^{(4n+1)\frac{\pi}{2}}$ b) $e^{(4n-1)\frac{\pi}{2}}$ c) $e^{(4n)\frac{\pi}{2}}$ d) $e^{-(4n+1)\frac{\pi}{2}}$	2	2

Q. No.	SECTION C $(2 \times 15 = 30)$ Answer ANY TWO questions	CO	KL
17.	a) If one root of the equation $x^3 + ax + b = 0$ is twice the difference of the other two, prove that one root is $\frac{13b}{3a}$. b) Find the equation whose roots exceed by 2, the roots of the equation $4x^4 + 32x^3 + 83x^2 + 76x + 21 = 0$. **(7+8)**	3	3
18.	a) Sum the series: $\left(1+\frac{1}{2}\right)+\left(\frac{1}{3}+\frac{1}{4}\right)\frac{1}{9}+\left(\frac{1}{5}+\frac{1}{6}\right)\frac{1}{9^2}+\cdots+\ \infty$. b) Express $\frac{\sin 6\theta}{\sin\theta}$ in terms of $\cos\theta$. **(7+8)**	3	3
19.	Diagonalise the following matrix: $\begin{bmatrix} 2 & -2 & 3 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{bmatrix}$.	3	3
20.	a) Express $\sin^7\theta\cos^3\theta$ as the sum of sines of multiples of θ. b) If $\cosh u = \sec\theta$ then show that $u = \log\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$. **(10+5)**	3	3

Q. No.	SECTION D $(2 \times 15 = 30)$ Answer ANY TWO questions	CO	KL
21.	(a) Solve the equation $x^4 + 16x^3 + 83x^2 + 152x + 84 = 0$ by removing its second term. b) Show that the sum of the series $\frac{1}{2!}+\frac{1+2}{3!}+\frac{1+2+2^2}{4!}\ldots+\infty = \frac{1}{2}(e-1)^2$. **(8+7)**	4	4
22.	a) If α,β,γ are the roots of the equation $x^3 + qx + r = 0$, find the value of $\frac{1}{\beta+\gamma}+\frac{1}{\gamma+\alpha}+\frac{1}{\alpha+\beta}$. b) Find the eigen values and eigen vectors of the following matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$. **(8+7)**	4	4

23.	a) Prove that the equation $\frac{ah}{\cos\theta} - \frac{bk}{\sin\theta} = a^2 - b^2$ has four roots and that the sum of the four values of θ which satisfy it is equal to an odd multiple of π radians. b) If $A = \begin{pmatrix} 4 & 2 \\ 3 & 3 \end{pmatrix}$, then compute A^n in terms of A. **(8+7)**	4	4
24.	a) Determine a, b and c such that $\lim_{\theta \to 0} \frac{\theta(a + b\cos\theta) - c\sin\theta}{\theta^5} = 1$. b) Find the general value of $Log_{(-3)}(-2)$. **(9+6)**	4	4

Q. No.	SECTION E $(\mathbf{2 \times 10 = 20})$ Answer ANY TWO questions	CO	KL
25.	If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$ form the equation whose roots are $\alpha + \beta, \beta + \gamma, \gamma + \alpha$.	5	5
26.	Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$ and hence determine its inverse.	5	5
27.	a) Find $\sum_{n=1}^{\infty} \frac{(-1)^n n^2}{n!}$. b) If $\frac{\tan\theta}{\theta} = \frac{2524}{2523}$, show that the value of θ is approximately $1^\circ 58'$. **(5+5)**	5	5
28.	a) Express $\cosh^7\theta$ in terms of hyperbolic cosines of multiples of θ. b) If $\lambda^2 - 5\lambda - 2 = 0$ is the characteristic equation of the matrix $A = \begin{pmatrix} 1 & 3 \\ 2 & 4 \end{pmatrix}$, then find A^4. **(5+5)**	5	5

⅄⅄⅄⅄⅄⅄⅄⅄⅄⅄⅄