STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 86
(For candidates admitted from the academic year 2023 – 2024)

B.Sc. DEGREE EXAMINATION, NOVEMBER 2024
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : ALLIED CORE
PAPER : MATHEMATICAL STATISTICS-1
SUBJECT CODE : 23MT/AC/ST35
TIME : 3 HOURS MAX. MARKS: 100

Q. No.	**SECTION A $(5 \times 2 = 10)$** **Answer ANY FIVE questions**	CO	KL
1.	Define marginal probability distribution for a discrete random variable.	1	1
2.	If X and Y are independent random variables then show that $\varphi_{X+Y}(\omega) = \varphi_X(\omega).\varphi_Y(\omega)$.	1	1
3.	Derive the mean of Binomial distribution	1	1
4.	Define standard normal distribution	1	1
5.	Using the property of quadratic expression, prove that $\lvert Cov\ (X,Y)\rvert \leq \sigma_X.\sigma_Y$ and hence $\lvert r_{XY}\rvert \leq 1$	1	1
6.	State any two properties of regression coefficients	1	1

Q. No.	**SECTION B $(10 \times 1 = 10)$** **Answer ALL questions**	CO	KL
7.	The cumulative distribution function of a random variable is ______ a) $F(x) = P(X \leq x)$ b) $F(x) = P(X < x)$ c) $F(x) = P(X = x)$ d) $F(x) = P(X > x)$	2	2
8.	A continuous random variable X has probability density function $f(x) = e^{-x}, 0 < x < \infty$, then $P(X > 1)$ is _______. a) e b) e^{-1} c) 1 d) ∞	2	2
9.	Mean of a constant 'a' is ________ a) 0 b) a c) 1 d) a^2	2	2
10.	A random variable X with probability density function $f(x) = {}^1/_4$, $-2 < x < 2$, the moment generating function of X is given by _______. a) $\frac{e^{2t}+e^{-2t}}{4t}$ b) $\frac{e^{2t}-e^{-2t}}{4t}$ c) $\frac{e^{2t}+e^{-2t}}{2t}$ d) $\frac{e^{2t}-e^{-2t}}{2t}$	2	2

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11.	If a random variable X satisfies Poisson distribution with a mean value 3, then $P(X \geq 1)$ is ______. a) $1 - e^{2}$ b) $1 - e^{-2}$ c) $1 + e^{2}$ d) $1 + e^{-2}$	2	2
12.	In an experiment, positive and negative values are equally likely to occur. The probability of obtaining at most one negative value in 5 trials is ________. a) $1/32$ b) $5/32$ c) $6/32$ d) $2/32$	2	2
13.	For a standard normal variate, the value of mean is? a) ∞ b) 1 c) 0 d) not defined	2	2
14.	Normal distribution is symmetric about ________. a) Standard deviation b) Mean c) Variance d) Covariance	2	2
15.	Which of the following could not be the value of a correlation coefficient? a) -0.34 b) -1 c) 1.08 d) 1	2	2
16.	Which of the following techniques is used to predict the value of one variable on the basis of other variables? a) Correlation analysis b) Correlation Corefficient c) Regression Analysis d) Spearman's rank Correlation	2	2

Q. No.	**SECTION C** $(\mathbf{2 \times 15 = 30})$ **Answer ANY TWO questions**	**CO**	**KL**
17.	If the joint probability density function is given by $f(x,y) = 24y(1-x)$ in $0 \leq y \leq x \leq 1$, find $Var(X), Var(Y)$ and $Cov(X,Y)$ by applying the appropriate formulas.	3	3
18.	a) Out of 800 families with 4 children each, assuming equal probabilities for boys and girls, find how many families would be expected to have (i) 2 boys and 2 girls (ii) Atmost 2 girls (iii) Atleast 1 boy b) If X and Y are independent Poisson random variables, show that the conditional distribution of X, given the value of $(X+Y)$ is a binomial distribution. **(7+8)**	3	3

19.	Fit a normal distribution to the following frequency distribution by the method of areas and hence find the theoretical frequencies	3	3

x	125	135	145	155	165	175	185	195	205	Total
f	1	1	14	22	25	19	13	3	2	100

20.	a) Calculate the correlation coefficient between X and Y using standard deviations for the following data:	3	3

X	29	30	28	31	28	33	27	35	23	36
Y	21	29	27	27	22	29	20	28	18	29

b) Ten students got the following percentage of marks in Maths and Physics

Students	1	2	3	4	5	6	7	8	9	10
Marks in Maths	78	36	98	25	75	82	90	62	65	39
Marks in Physics	84	51	91	60	68	62	86	58	63	47

Calculate the rank correlation coefficient **(8 + 7)**

Q. No.	**SECTION D ($\mathbf{2 \times 15 = 30}$)** **Answer ANY TWO questions**	CO	KL
21.	A continuous random variable X has the p.d.f, $f(x) = kx^2$, $0 \le x \le 1$. Find (i) the value of k (ii) $P\left(\frac{1}{4} < X < \frac{1}{2}\right)$ (iii) $P\left(X > \frac{3}{4} / X > \frac{1}{2}\right)$ (iv) The value of a such that $P(X \le a) = P(X > a)$ (v) The value of b such that $P(X > b) = 0.05$	4	4
22.	a) If the probability density function of a continuous random variable X is given by $f(x) = k(1+x)e^{-2x}$, in $-1 \le x \le \infty$. Analyze the procedure to find the value of k, mean and variance of X by finding the moment generating function of X. b) Analyze the relationships between the first four cumulants and their corresponding central moments. **(9 + 6)**	4	4
23.	a) If X is a discrete random variable following a Poisson distribution with parameter λ, show that $P(X = \text{even}) = e^{-\lambda}\cosh\lambda$ and $P(X = \text{odd}) = e^{-\lambda}\sinh\lambda$ b) Find the probability that at most 5 defective fuses will be found in a box of 200 fuses, if experiences shows that 2% of such fuses are defective. **(8+7)**	4	4
24.	A number X is chosen randomly from the integers 1,2,3,4 and a number Y is chosen from among those at least as large as X. Find the equations of the regression line of Y on X and that of X on Y.	4	4

Q. No.	**SECTION E** $(\mathbf{2} \times \mathbf{10} = \mathbf{20})$ **Answer ANY TWO questions**	**CO**	**KL**
25.	Three balls are drawn at random without replacement from a box containing 2 white, 3 red and 4 black balls. If X denotes the number of white balls drawn and Y denotes the number of black balls drawn, find the joint probability distribution of (X, Y).	5	5
26.	State and prove Tchebycheff's Inequality.	5	5
27.	The marks obtained by a number of students in a certain subject are approximately normally distributed with mean 65 and standard deviation 5. If 3 students are selected at random from this group, what is the probability that at least one of them would have scored above 75?	5	5
28.	Find the standard errors of estimate of Y on X and X on Y from the following data:	5	5

X	1	2	3	4	5
Y	2	5	9	13	14

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