**STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086**
**(For candidates admitted during the academic year 2023 – 24 & thereafter)**

**B. Sc. DEGREE EXAMINATION, NOVEMBER 2024**
**BRANCH III - PHYSICS**
**FIRST SEMESTER**

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|---|---|---|
| **COURSE** | **:** | **ALLIED – CORE** |
| **PAPER** | **:** | **MATHEMATICS FOR PHYSICS – I** |
| **SUBJECT CODE** | **:** | **23MT/AC/MP15** |
| **TIME** | **:** | **3 HOURS** |

**MAX. MARKS : 100**

| Q.NO. | SECTION A (5×2=10)<br>Answer any FIVE questions | CO | KL |
|---|---|---|---|
| 1 | State Cayley Hamilton's theorem. | 1 | 1 |
| 2 | Write the characteristic equation of a square matrix A. | 1 | 1 |
| 3 | Find the $n^{th}$ differential co-efficient of $x^2e^{3x}$. | 1 | 1 |
| 4 | Eliminate the arbitrary function from $z = f(x^2 + y^2)$. | 1 | 1 |
| 5 | Define odd and even function and give examples. | 1 | 1 |
| 6 | Define feasible solution. | 1 | 1 |

| Q.NO. | SECTION B (10×1=10)<br>Answer ALL questions | CO | KL |
|---|---|---|---|
| 7 | The eigen values of $A = \begin{bmatrix} 7 & 0 \\ 0 & 1 \end{bmatrix}$ are,<br>a) $-1, 7$; b) $1, -7$; c) $-1, -7$; d) $1, 7$. | 2 | 2 |
| 8 | If A and B have the same characteristic equation then, they are,<br>a) Singular; b) Similar; c) Unitary; d) Symmetric. | 2 | 2 |
| 9 | If $y = sin2x$ then $y_2 = ?$<br>a) $4\sin(\pi + 2x)$ ; b) $2\sin(\pi + 2x)$ ; c) $4\sin\left(\frac{\pi}{2} + 2x\right)$ ; d) $2\sin(\frac{\pi}{2} + 2x)$. | 2 | 2 |
| 10 | If $y = e^{3x+5}$ then $y_n = ?$<br>a) $(3x + 5)e^{3x+5}$ ; b) $3^n e^{3x+5}$ ; c) $(3^n + 5)e^{3x+5}$ ; d) $3^n e^{3x}$. | 2 | 2 |
| 11 | The solution to the PDE $pq = 1$ is $z = ?$<br>a) $ax + by + c$ ; b) $ax + \frac{y}{b} + c$ ; c) $ax + \frac{y}{a} + c$ ; d) $ax + by + ab$. | 2 | 2 |
| 12 | The solution to the Clairaut's form PDE $z = px + qy + pq$ is $z = ?$<br>a) $ax + by + c$ ; b) $bx + ay + c$ ; c) $x + y + 1$ ; d) $ax + by + ab$. | 2 | 2 |
| 13 | If $f(x) = \pi - x,\ 0 < x < 2\pi$, then in its Fourier series expansion $a_0 = ?$<br>a) $2\pi$ ; b) $\pi$ ; c) $\frac{\pi}{2}$ ; d) 0. | 2 | 2 |
| 14 | If $f(x) = x,\ -\pi < x < \pi$, then in its Fourier series expansion $a_n = ?$<br>a) 0 ; b) $(-1)^n$ ; c) $2(-1)^n$ ; d) $\frac{2(-1)^n}{n}$. | 2 | 2 |
| 15 | The minimum value of the objective function $Z = 3x + 2y$ when $x = 1, y = 5$ is,<br>a) 20 ; b) 13 ; c) 16 ; d) 36. | 2 | 2 |
| 16 | The extreme values of the objective function $Z = 3x + 4y$ are (20,0), (30.8,11.5), (0,30), (0,25) then the maximum of $Z$ is,<br>a) 60 ; b) 138.4 ; c) 120 ; d) 100. | 2 | 2 |

| Q.NO. | **SECTION C (2×15=30)** **Answer any TWO questions** | CO | KL |
|---|---|---|---|
| 17 | Evaluate the matrix $A^6 - 25A^2 + 122A$, where $A = \begin{bmatrix} 0 & 0 & 2 \\ 2 & 1 & 0 \\ -1 & -1 & 3 \end{bmatrix}$. | 3 | 3 |
| 18 | Integrate the following: i) $\sqrt{\frac{5-x}{x-2}}$ ; ii) $\frac{1}{(3+x)\sqrt{x}}$. (7+8) | 3 | 3 |
| 19 | Solve the following PDE (i) $q = xp + p^2$ ; (ii) $9(p^2z + q^2) = 4$. (7+8) | 3 | 3 |
| 20 | Show that $x^2 = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty}(-1)^n \frac{cosnx}{n^2}$ in the interval $-\pi \leq x \leq \pi$ and deduce that $\frac{\pi^2}{12} = \frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} \dots$ | 3 | 3 |

| Q.NO. | **SECTION D (2×15=30)** **Answer any TWO questions** | CO | KL |
|---|---|---|---|
| 21. | Diagonalize the matrix $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$. | 4 | 4 |
| 22. | If $I_n = \frac{d^n}{dx^n}(x^n logx)$, prove that $I_n = nI_{n-1} + (n-1)!$ And hence, show that $I_n = n!\left\{logx + 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}\right\}$. | 4 | 4 |
| 23. | Find the general solution of $(y + z)p + (z + x)q = x + y$. | 4 | 4 |
| 24. | Solve the following LPP: $Max\ Z = 3x_1 + 2x_2 + 5x_3\ subject\ to\ x_1 + 2x_2 + x_3 \leq 430;\ 3x_1 + 2x_3 \leq 460;\ x_1 + 4x_2 \leq 420;\ x_1, x_2, x_3 \geq 0.$ | 4 | 4 |

| Q.NO. | **SECTION E (2×10=20)** **Answer any TWO questions** | CO | KL |
|---|---|---|---|
| 25. | If $y = \sin(msin^{-1}x)$ show that $(1 - x^2)y_2 - xy_1 + m^2y = 0$ | 5 | 5 |
| 26. | Solve the following PDE (i) $p = y^2q^2$ ; (ii) $py + qx = pq$. (5+5) | 5 | 5 |
| 27. | Express $f(x)$ as a Fourier series in $-\pi < x < \pi$ , and hence deduce that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$ where $f(x) = \begin{cases} -x, & -\pi < x < 0 \\ x, & 0 < x > \pi \end{cases}$ | 5 | 5 |
| 28. | Solve by graphical method: $Min\ Z = 20x + 10y\ subject\ to$ $x + 2y \leq 40; 3x + y \geq 30; 4x + 3y \geq 60; x, y \geq 0.$ | 5 | 5 |

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