STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019 – 20 & thereafter)

B. Sc. DEGREE EXAMINATION, NOVEMBER 2024
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE : MAJOR – CORE
PAPER : VECTOR ANALYSIS AND APPLICATION
SUBJECT CODE : 19MT/MC/VA53
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(10×2=20)**
ANSWER ANY TEN QUESTIONS

1. Define scalar field and give an example.
2. If $\phi(x, y, z) = xy^3 + 3xz^2$ then find grad ϕ at the point (1, -2, 3).
3. Prove that $curl\,\vec{r} = 0$ where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$.
4. Determine the constant a so that the vector $\vec{V} = (2x + 3y)\,\hat{\imath} + (3y - 2z)\hat{\jmath} + (x + az)\,\hat{k}$ is solenoidal.
5. Calculate the work done by the force $\vec{F} = 2y\,\hat{\imath} + xy\,\hat{\jmath}$ lb in moving an object along a straight line from $A(0, 0, 0)$ to $B(2, 1, 0)$ ft.
6. If S is any closed surface enclosing a volume V and $\vec{F} = x\hat{\imath} + 2y\hat{\jmath} + 3z\hat{k}$ then prove that $\iint_S \vec{F}.\hat{n}\, dS = 6V$.
7. Define Laplace's equation.
8. If $\vec{f} = (x + y + 1)\hat{\imath} + \hat{\jmath} + (-x - y)\hat{k}$ then prove that $\vec{f} \cdot curl\,\vec{f} = 0$.
9. Find the unit normal to the surface $x^2 - y^2 + z = 2$ at the point $(1, -1, 2)$.
10. Define orthogonal curvilinear coordinate system.
11. Find the unit tangent vector to any point on the curve $x = a\cos t,\ y = a\sin t,\ z = bt$.
12. Prove that $\oint_C \vec{r} \cdot d\vec{r} = 0$.

SECTION – B **(5×8=40)**
ANSWER ANY FIVE QUESTIONS

13. If $\vec{A}(t) = 3t^2\,\hat{\imath} - (t + 4)\hat{\jmath} + (t^2 + 2t)\hat{k}$ and $\vec{B}(t) = 2\sin t\ \hat{\imath} + 3e^{-t}\,\hat{\jmath} + 3\cos t\,\hat{k}$ then find $\frac{d}{dt}(\vec{A} \times \vec{B})$ at $t = 0$.
14. Prove that $curl\ grad\ \phi = 0$ and $div\ curl\ \vec{A} = 0$ where $\vec{A} = A_1\,\hat{\imath} + A_2\,\hat{\jmath} + A_3\,\hat{k}$.
15. Find the values of the constants a, b, c so that the directional derivative $\phi = axy^2 + byz + cz^2x^3$ at (1, 2, -1) has maximum of magnitude 64 in a direction parallel to z-axis.
16. Evaluate by Stoke's theorem $\oint_C (e^x\,dx + 2y\,dy - dz)$ where C is the curve $x^2 + y^2 = 4, z = 2$.

17. Find the total work done in moving a particle in a force field given by $\vec{F} = 3xy\,\hat{\imath} - 5z\,\hat{\jmath} + 10x\,\hat{k}$ along the curves $x = t^2 + 1,\ y = 2t^2,\ z = t^3$ from $t = 1$ to $t = 2$.
18. Discuss physical interpretation of curl.
19. If f and g are two scalar point functions the prove that $\nabla\left(\frac{f}{g}\right) = \frac{g\,\nabla f - f\,\nabla g}{g^2}$.

SECTION – C **(2×20=40)**

ANSWER ANY TWO QUESTIONS

20. a) If $r = |\vec{r}|$ where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ then prove that (i) $\nabla \log|\vec{r}| = \frac{\vec{r}}{r^2}$ and (ii) $\nabla r^n = nr^{n-2}\vec{r}$.

    b) If $\vec{A} = x^2z\,\hat{\imath} + yz^3\hat{\jmath} - 3xy\,\hat{k}$, $\vec{B} = y^2\,\hat{\imath} - yz\,\hat{\jmath} + 2x\,\hat{k}$ and $\phi(x,y,z) = 2x^2 + yz$ then find (i) $\vec{A}\cdot(\nabla\phi)$ and (ii) $(\vec{A}\cdot\nabla)\vec{B}$. (10 + 10)

21. a) If $\vec{w}$ is a constant vector and $\vec{v} = \vec{w} \times \vec{r}$ where $\vec{r} = x\hat{\imath} + y\hat{\jmath} + z\hat{k}$ and $\vec{w} = w_1\hat{\imath} + w_2\,\hat{\jmath} + w_3\hat{k}$. Prove that (i) $\vec{w} = \frac{1}{2} curl\ \vec{v}$ and (ii) $div\ \vec{v} = 0$.

    b) Verify Green's theorem in the plane for $\oint_C (xy + y^2)dx + x^2dy$ where C is the closed curve of the region bounded by $y = x$ and $y = x^2$. (8 + 12)

22. a) Derive divergence in terms of curvilinear coordinates.

    b) Prove that $r^n\vec{r}$ is solenoidal only when $n + 3 = 0$. (12 + 8)