STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019 – 20 & thereafter)

B. Sc. DEGREE EXAMINATION, NOVEMBER 2024
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE : MAJOR – CORE
PAPER : PRINCIPLES OF REAL ANALYSIS
SUBJECT CODE : 19MT/MC/RA55
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(10 × 2 = 20)**
ANSWER ANY TEN QUESTIONS

1. Define limit of a function on the real line.
2. Prove that $\lim_{x\to\infty}\left(\frac{1}{x^2}\right) = 0$.
3. Find $B[a\,;1]$, $B[a\,;2]$ in the real line with the discrete metric.
4. Prove that every subset of a discrete metric space is open.
5. Show with suitable example that any subset of a metric space need not contain all its limit points.
6. If $|x - 2| < 1$, prove that $|x^2 - 4| < 5$.
7. Check whether the interval [0, 1] is a connected subset of a discrete metric space.
8. Give an example of a set which is bounded and totally bounded.
9. Define compact metric space.
10. Define uniform continuity of a function from one metric space into another metric space.
11. Define Upper sum and Lower sum.
12. Verify whether the integral $\int_1^\infty \frac{1}{x}\,dx$ converges or diverges?

SECTION – B **(5 × 8 = 40)**
ANSWER ANY FIVE QUESTIONS

13. If f is a real-valued function continuous at $a \in \mathbb{R}^1$ if and only if $\lim_{n\to\infty} x_n = a$ implies $\lim_{n\to\infty} f(x_n) = f(a)$, where $\{x_n\}_{n=1}^{\infty}$ is a sequence of real numbers.
14. Let (M, ρ) be a metric space and let a be a point in M. Let f and g be real-valued functions whose domains are subsets of M. If $\lim_{n\to\infty} f(x) = L$ and $\lim_{n\to\infty} g(x) = N$ then prove that $\lim_{n\to\infty}[f(x)g(x)] = LN$.
15. Prove that if G_1 and G_2 are open subsets of the metric space M, then $G_1 \cap G_2$ is also open.
16. State and prove nested interval theorem.
17. Let f be a continuous function from the compact metric space M_1 into the metric space M_2 then prove that the range $f(M_1)$ of f is compact.
18. Prove that differentiability at a point implies continuity at a point. Is the converse true, justify with an example.
19. State and prove Rolle's theorem.

SECTION – C **($2 \times 20 = 40$)**

ANSWER ANY TWO QUESTIONS

20.(a) If f and g are real-valued functions which are continuous at a and $f(a)$ respectively then prove that $g \cdot f$ is continuous at a.

(b) Prove that ℓ^{∞} which is the set of all bounded sequences of real numbers is a metric space by defining a suitable metric. (10 + 10)

21.(a) If $\mathcal{F}$ is any family of closed subsets of a metric space M then prove that $\bigcap_{F\in\mathcal{F}} F$ is closed.

(b) If the subset A of the metric space $< M, \rho >$ is totally bounded then prove that A is bounded set. Is the converse true? Justify with an example. (8 + 12)

22. (a) If the metric space M has the Heine-Borel property then prove that M is compact.

(b) Let $f(x) = x$ $(0 \le x \le 1)$ and for each $n \in I$ let σ_n be the subdivision $\left\{0, \frac{1}{n}, \frac{2}{n}, \cdots, \frac{n}{n}\right\}$ of $[0,1]$. Compute $\lim_{n\to\infty} U[f;\sigma_n]$ and $\lim_{n\to\infty} L[f;\sigma_n]$. (12 + 8)