

**STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086**  
**(For candidates admitted during the academic year 2019 – 20 & thereafter)**

**B. Sc. DEGREE EXAMINATION, NOVEMBER 2024**  
**BRANCH I - MATHEMATICS**  
**FIFTH SEMESTER**

**COURSE : MAJOR – CORE**  
**PAPER : INTEGRAL TRANSFORMS**  
**SUBJECT CODE : 19MT/MC/IT54**  
**TIME : 3 HOURS** **MAX. MARKS : 100**

**SECTION – A** **(10×2=20)**  
**ANSWER ANY TEN QUESTIONS**

1. Define Laplace transform.
2. Define periodic function.
3. State any two applications of Laplace Transforms.
4. Find  $L^{-1}\left(\frac{s}{s^2+k^2}\right)$ , where  $k$  is a constant.
5. State Fourier Integral theorem.
6. Find the Fourier sine transform of  $\frac{1}{x}$ .
7. If  $F\{f(x)\}$ , then find the transform  $F\{e^{iax}f(x)\}$ .
8. Find the  $Z$  – transform of unit impulse function.
9. Compute the transform  $Z\{e^{-at}\}$ .
10. State the linearity property of  $Z$  – transform.
11. Evaluate  $Z(n)$  using inverse  $Z$  – transform for the function  $\frac{z}{(z-1)^2}$ .
12. Write down the inverse  $Z$  – transform of  $F(z)$  formula in Integral form.

**SECTION – B** **(5×8=40)**  
**ANSWER ANY FIVE QUESTIONS**

13. Evaluate  $L^{-1}\left(\frac{s-3}{s^2+4s+13}\right)$ .
14. Evaluate  $\int_0^{\infty} \frac{e^{-t}-e^{-2t}}{t} dt$  using Laplace transforms.
15. Solve  $\frac{d^2y}{dt^2} + 4\frac{dy}{dt} - 5y = 5$ , given that  $y = 0, \frac{dy}{dt} = 2$  when  $t = 0$  using Laplace transforms.
16. Find the Fourier transform of  $f(x) = \begin{cases} 1, & |x| < 1 \\ 0, & |x| > 1 \end{cases}$ . Hence evaluate  $\int_0^{\infty} \frac{\sin x}{x} dx$ .
17. Prove that the Fourier transform of  $e^{-\frac{x^2}{2}}$  is self-reciprocal.
18. Find  $Z\left[\frac{2n+3}{(n+1)(n+2)}\right]$ .
19. Evaluate the inverse  $Z$ -transform of  $\left[\frac{z}{(z-1)^2(z+1)}\right]$  by partial fraction method.

**SECTION – C**  
**ANSWER ANY TWO QUESTIONS**

**(2×20=40)**

20. a) If  $L\{f(t)\} = F(s)$ , then prove that  $L\{tf(t)\} = -\frac{d}{ds}F(s)$  and hence find  $L\{te^{-t}\sin t\}$ .
- b) Determine  $y$  which satisfies the equation  $\frac{dy}{dt} + 3y + 2\int_0^t y dt = t$  for which  $y(0) = 0$  using Laplace transforms. (10+10)
21. a) Using Fourier integral, evaluate  $\int_0^\infty \frac{\cos tx}{1+t^2} dx$ .
- b) State and prove the final value theorem in Z transforms. (10+10)
22. a) Solve  $y_{n+2} - 4y_{n+1} + 4y_n = 0$  given  $y_0 = 1, y_1 = 0$  using Z transforms.
- b) Solve the following simultaneous equations by using Laplace transform:  
 $3\frac{dx}{dt} + \frac{dy}{dt} + 2x = 1; \frac{dx}{dt} + 4\frac{dy}{dt} + 3y = 0$  given  $x(0) = 0 = y(0)$ . (10+10)

