STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019–20 & thereafter)

B. Sc. DEGREE EXAMINATION, NOVEMBER 2024
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE	**:**	**MAJOR – CORE**
PAPER	**:**	**ALGEBRAIC STRUCTURES**
SUBJECT CODE	**:**	**19MT/MC/AS55**
TIME	**:**	**3 HOURS**

MAX. MARKS : 100

SECTION – A

Answer any ten questions: **(10 × 2 =20)**

1. Show that for each element *a* in a group *G*, there is a unique element *b* in *G* such that *ab* = *ba* = *e*.
2. List the subgroups of $\mathbb{Z}_{30}$.
3. Express the permutation (153)(246)(352) as a product of disjoint cycles form and hence rewrite it as a product of 2-cycles.
4. Define inner automorphism induced by an element *a* in group *G*.
5. Find all the distinct left cosets of $(3Z, +)$which is subgroup of the group $(Z, +)$.
6. Find the characteristic of ring of integers and $\mathbb{Z}_n$.
7. Show that a group *G* having a unique subgroup *H* of finite order is normal in *H*.
8. Define group homomorphism and give an example.
9. Show that the ideal $\langle x^2 + 1\rangle$ is maximal in *R*[*x*].
10. Is Z_m a homomorphic image of *Z*? Justify.
11. Define isometry in R^n.
12. State any two consequences of Lagrange's theorem.

SECTION – B

Answer any five questions: **(5 × 8 = 40)**

13. State and prove one-step subgroup test.

14. (a) Determine whether the given permutation (12)(134)(152) is odd or even.
    (b) Express the permutation $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 2 & 3 & 4 & 5 & 1 & 7 & 8 & 6 \end{pmatrix}$ as product of 2-cycles. (3+5)

15. State and prove Cayley's theorem.

16. (a) Define orbit and stabilizer of a point for a group of permutations.
    (b) Determine the orbit and stabilizer of a point for the group
    G = {(1), (132)(465)(78), (132)(465), (123)(456), (123)(456)(78), (78)}. (4+4)

17. (a) Give an example of a subset of a ring that is a subgroup under addition but not a subring.
    (b) State and prove subring test. (2+6)

18. (a) Illustrate with an example the factor group concept for $G = Z_{18}$ and *H* = ⟨6⟩.
    (b) State and prove the normal subgroup test. (3+5)

19. State and prove the theorem on existence of factor rings.

SECTION – C

Answer any two questions: **($2 \times 20 = 40$)**

20. (a) Prove that every permutation of a finite set can be written as a cycle or as a product of disjoint cycles.
    (b) State and prove fundamental theorem of cyclic groups. (8 + 12)

21. (a) If *G* is a group and *H* is a normal subgroup of *G* then prove that the set $G/H = \{aH : a \in G\}$ is a group under the operation $(aH)(bH) = abH$.
    (b) Prove that kernel of a homomorphism is a normal subgroup of the group.
    (c) State and prove first Isomorphism theorem for groups and explain with an example on wrapping function. (6+ 4+10)

22. (a) Prove that *R*/*A* is a field if and only if *A* is maximal where *R* is a commutative ring with unity and *A* is an ideal of *R*.
    (b) Show that there exists a field *F* (called the field of quotients of *D*) that contains a subring isomorphic to *D*. (8+12)