

B. Sc. DEGREE EXAMINATION, NOVEMBER 2012
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE : MAJOR – ELECTIVE

PAPER : OPTIMIZATION TECHNIQUES

TIME : 3 HOURS

MAX. MARKS : 100

Answer Any Six Questions (each carrying 17marks)

1. Use Golden Mean Search to approximate the location of the maximum of $f(x) = x(5\pi - x)$ on $[0,20]$ to within $\varepsilon = 1$.
2. Use the Newton Raphson Method to maximize $z = -(x_1 - \sqrt{5})^2 - (x_2 - \pi)^2 - 10$ to within a tolerance of 0.05.
3. Use Lagrange Multipliers to minimize $z = x_1 + x_2 + x_3$ subject to $x_1^2 + x_2 = 3$ and $x_1 + 3x_2 + 2x_3 = 7$.
4. Use the method of feasible directions to maximize $z = x_1 + x_2$ subject to $x_2x_1 - 2x_2 \leq 3$, $3x_1 + 2x_2 \leq 24$, and $x_1, x_2 \geq 0$.
5. A presidential nominee has reduced the field of possible Vice presidential running mates to three people. Each of these candidates has been rated on a scale from 1 (lowest) to 10 (highest). Person 1 received 10 points, person 2 received 8 points and person 3 received 5 points. The probability of person i ($i = 1,2,3$) accepting the j^{th} ($j = 1,2,3$) offer to run for Vice President (assuming the first $j - 1$ offers, to other people, were declined) is denoted by p_{ij} where
$$p_{11} = 0.5 \quad p_{12} = 0.2 \quad p_{13} = 0$$
$$p_{21} = 0.9 \quad p_{22} = 0.5 \quad p_{23} = 0.2$$
$$p_{31} = 1 \quad p_{32} = 0.8 \quad p_{33} = 0.4$$
In what order should the three potential running mates be offered the Vice Presidential nomination if the presidential nominee wants to maximize the expected number of points?
6. a) Explain Markov Process.
b) Is the stochastic matrix $P = \begin{pmatrix} 0 & 1 \\ 0.4 & 0.6 \end{pmatrix}$ regular? ergodic?
Calculate $L = \lim_{n \rightarrow \infty} P^n$ if it exists.

7. A new Television set arrives for inspection every 3 min and is taken by a quality control Engineer on a first come, first second basis. There is only one engineer on duty and it takes exactly 4 min to inspect each new set. Determine the average number of sets waiting to be inspected over the first half hour of a shift, if there are no sets awaiting inspection at the beginning of the shift.

8. A gourmet delicatessen is operated by one person, the owner. The arrival pattern of customers on Saturdays appears to follow Poisson distribution, with a mean arrival rate of 10 people per hour. Customers are served on a FIFO basis and because of the reputation of the store they are willing to wait for service once they arrive. The time it takes to serve a customer is estimated to be exponentially distributed, with an average service time of 4 min. Determine (a) the probability that there is a queue. (b) the average size of the queue. (c) the expected time that a customer must wait in the queue and (d) the probability that a customer will spend less than 12 min in the store.

