STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2008–09 & thereafter)

SUBJECT CODE : MT/ME/FM54

B. Sc. DEGREE EXAMINATION, NOVEMBER 2012
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE : MAJOR – ELECTIVE
PAPER : FINANCIAL MATHEMATICS
TIME : 3 HOURS **MAX. MARKS : 100**

Answer Any Six Questions (each carrying 17marks)

1. (a) Can you construct a pair of random variable such that $Var(X) = Var(Y) = 1$ and $Cov(X, Y) = 2$? Justify your answer. (2)

   (b) Define Geometric Brownian motion and show that this motion is a limit of simpler models. (15)

2. (a) Find the yield curve and the present value function if $r(s) = \frac{r_1}{1+s} + \frac{s\,r_2}{1+s}$. (5)

   (b) The annual rainfall in Cleveland, Ohio, is normally distributed with mean 40.14 inches and standard deviation 8.7 inches. Find the probability that the sum of the next two years rainfall exceeds 84 inches. (Given that $\emptyset(.30) = .6179$) (5)

   (c) Suppose that you are to receive payments (in thousands of dollars) at the end of each of the next five years. Which of the following three payment sequences is preferable if the interest rate compounded annually at 20%?

   A : 12, 14, 16, 18, 20 B : 16, 16, 15, 15, 15 C : 20, 16, 14, 12, 10. (7)

3. (a) Illustrate 'option pricing' by an example. (9)

   (b) Let $C(K, t)$ be the cost of a call option on a specified security that has strike price *K* and expiration time *t*. Prove that (a) For fixed expiration time *t*, $C(K, t)$ is a convex in *K* and non-increasing function of *K* and (b) For $s > 0, C(K, t) - c(K + s, t) \leq s$. (8)

4. (a) State and prove Arbitrage Theorem. (10)

   (b) State and prove the put-call option parity formula. (7)

5. Derive the Black-Scholes option cost, $C = s\phi(\omega) - Ke^{-rt}\phi(\omega - \sigma\sqrt{t})$. (17)

6. (a) Prove that the amount of money needed at time 0 is equal to the expected present value, under the risk-neutral probabilities of the payoff at time 1, by delta hedging arbitrage strategy. (12)

(b) Suppose that a security is presently selling for a price of 30, the nominal interest rate is 8% (with the unit of time being one year) and the security's volatility is 0.20. Find the no-arbitrage cost of a call option that expires in three months and has a strike price of 34. (5)

7. (a) Solve the following Problem.

Maximize $E[U(W)]$

Subject to

$\sum_{i=1}^{n} \omega_i = \omega \;\; ; \quad \omega_i \geq 0, i = 1,2, \dots, n.$

where U is the investor's utility function. (10)

(b) An investor with capital x can invest any amount between 0 and x; if y is invested then y is either won or lost, with respective probabilities p and $1 - p$. If $p > 1/2$, how much should be invested by an investor having a log utility function? (7)

8. (a) Obtain 'value at risk' for normal random variable. (5)

(b) Explain in details Barrier, Asian and Look back Options. (12)

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