

STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2008–09 & thereafter)

SUBJECT CODE : MT/MC/RD54

B. Sc. DEGREE EXAMINATION, NOVEMBER 2012
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE : MAJOR – CORE
PAPER : RANDOM VARIABLES AND THEORETICAL DISTRIBUTION
TIME : 3 HOURS **MAX. MARKS : 100**

ANSWER ANY SIX QUESTIONS: (EACH QUESTION CARRIES EQUAL MARKS)

1. (a) What do you mean by independence of events.
(b) State and prove the addition theorem of probability.
(c) State and prove Baye's theorem on inverse probability. (2+7+8)

2. (a) The odds that person X speaks the truth are 3:2 and the odds that person Y speaks the truth are 5:3. In what percentage of cases are they likely to contradict each other on an identical point?
(b) Define conditional probability.
(c) State and prove Chebychev's inequality. (7+2+ 8)

3. (a) State the properties of distribution function.
(b) Prove that two random variables X and Y with joint p.d.f $f(x, y)$ are stochastically independent if and only if $f_{x,y}(x, y)$ can be expressed as the product of a non-negative function of x alone and a non – negative function of y , i.e., if $f_{x,y}(x, y) = f_x(x)f_y(y)$.
(c) The joint probability distribution of two random variables X and Y are given by:
 $P(X = 0, Y = 1) = 1/3$, $P(X = 1, Y = -1) = 1/3$ and $P(X = 1, Y = 1) = 1/3$. Find
(i) marginal distributions of X and Y and (ii) the conditional probability distribution of X given $Y=1$. (5+6+6)

4. (a) Given $f(x, y) = cx(x - y), 0 < x < 2, -x < y < x$
 0 otherwise

Find c , $f(x)$ and $f(x/y)$.

- (b) Explain marginal and conditional density functions.
 (c) Let X and Y be jointly distributed with p. d. f:

$$f_{x,y}(x, y) = \begin{cases} \frac{1}{4}(1 + xy) & |x| < 1, |y| < 1 \\ 0 & \text{otherwise} \end{cases}$$

Show that X and Y are not independent but X^2 and Y^2 are independent. (6+4+7)

5. (a) Define mathematical expectation.
 (b) State and prove the multiplication theorem of expectation.
 (c) Define M.G.F. State and prove its properties. (2+6 +9)

6. (a) Let X and Y be two random variables, having joint density function

$$f(x, y) = \begin{cases} \frac{6-x-y}{8}, & 0 \leq x \leq 2, 2 \leq y \leq 4 \\ 0 & \text{otherwise} \end{cases}$$

Find the correlation between X and Y .

- (b) State Uniqueness theorem on characteristic function. (15+2)

7. (a) State & prove additive property for binomial distribution.
 (b) Find the moment generating function of Poisson distribution and find measure of skewness and measure of kurtosis.
 (c) Obtain points of inflexion of normal curve. (6+8+3)

8. (a) Find the mode of Normal distribution.
 (b) Let X and Y are independent Poisson variates, prove that the conditional distribution of X given $X+Y$ is Binomial.
 (c) Find the moment generating function normal distribution and hence prove that it is symmetric. (5+5+7)

