STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2008 – 09 & thereafter)

SUBJECT CODE : MT/MC/RA54

B. Sc. DEGREE EXAMINATION, NOVEMBER 2012
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE : MAJOR – CORE
PAPER : REAL ANALYSIS
TIME : 3 HOURS **MAX. MARKS : 100**

ANSWER ANY SIX QUESTIONS

1. a) Compute $\underset{x\to 3}{\text{Lt}}\ x^2 + 2x = 15$ from the first principles.
   b) If $\underset{x\to a}{\text{Lt}}\ f(x) = L$ and $\underset{x\to a}{\text{Lt}}\ g(x) = M,$ then prove:
   (i) $\underset{x\to a}{\text{Lt}}\ [f(x) + g(x)] = L + M.$
   (ii) $\underset{x\to a}{\text{Lt}}(1/g(x)) = 1/M,$ if $M \neq 0.$ (7+10)

2. a) If f and g are real-valued functions which are continuous at $a \,\epsilon\, \mathfrak{R}$, then prove: (i) $f + g$ is continuous at a.
   (ii) fg is continuous at a.
   b) Prove that if f is continuous at a, g is continuous at $f(a)$, then gof is continuous at a, where f, g are real valued functions defined on $\mathfrak{R}'$. (10+7)

3. (a) Prove that in the complex plane C, if $Z_n = 1 + n^{-2} + (2 - 1/n)i$, then $Z_n \to 1 + 2i$.
   (b) Show that the limit of a convergent sequence in a metric space is unique.
   (c) In any metric space (S, d_s) prove that every compact subset T is complete.
   (5+6+6)

4. (a) Let (S, d_s) and (T, d_T) be two metric spaces and let $f: S \to T$ and $g: f(S) \to U$ be functions and let h be the composite function defined by the equation $h(x) = f(g(x))$, $x \,\epsilon\, S$. Prove that if f is continuous at p and g is continuous at $f(p)$, then h is continuous at p.
   (b) Prove that a continuous image of a compact set is compact.
   (c) Let $f: S \to T$ be a function from one metric space (S, d_s) to another metric space (T, d_T). Show that f is continuous on S if for every closed set Y in T, $f^{-1}(Y)$ is closed in S. (6+5+6)

5. (a) Define homeomorphism, topological property. Give an example for each.
   (b) State and prove Bolzano's Theorem.
   (c) Prove that a metric space S is connected if every two valued function on S is a constant. (4+8+5)

6. (a) Define arcwise connectedness and prove that every arcwise connected set in $\mathfrak{R}^n$ is connected.
   (b) State and prove the Fixed Point Theorem. (7+10)

7. (a) If $f(x) = x$, $0 \leq x \leq 1$ and if σ be the subdivision {0, 1/3, 2/3, 1}, compute $U(f, \sigma)$ and $L(f, \sigma)$.
   (b) If $f \in \mathfrak{R}[a, b]$, and λ is any real number then prove $\lambda f \in \mathfrak{R}[a, b]$ and $\int_a^b \lambda f = \lambda \int_a^b f$.
   (c) If $f, g \in \mathfrak{R}[a, b]$, then prove that $f + g \in \mathfrak{R}[a, b]$ and $\int_a^b f + g = \int_a^b f + g$. (4+6+7)

8. (a) State and prove the Chain Rule for differentiation.
   (b) State and prove the Generalised Mean Value Theorem.
   (c) Verify Rolle's Theorem for the function $f(x) = 2 + (x-1)^{2/3}$ on [0,2]. (6+6+5)