STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted during the academic year 2011–12)

SUBJECT CODE: 11MT/MC/VA34

B. Sc. DEGREE EXAMINATION, NOVEMBER 2012
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : MAJOR – CORE
PAPER : VECTOR ANALYSIS AND ITS APPLICATIONS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION-A

Answer All the questions **(10 x 2 = 20)**

1. If $\vec{R} = \sin t\vec{i} + \cos t\vec{j} + t\vec{k}$ then find $\frac{d\vec{R}}{dt}$ and $\frac{d^2\vec{R}}{dt^2}$.
2. Find the unit tangent vector to any point on the curve $x = t^2 + 1, y = 4t - 3, z = 2t^2 - 6t$
3. Show that the vector $\vec{F} = z\vec{i} + x\vec{j} + y\vec{k}$ is solenoidal.
4. Prove that $curl\,\vec{r} = 0$ where $\vec{r}$ is the position vector of the point (x,y,z).
5. If $\vec{F} = (x^2 - y^2 + 2xz)\vec{i} + (xz - xy + yz)\vec{j} + (z^2 + x^2)\vec{k}$,find $curl\,\vec{F}$ at (1, 1, 1).
6. If $\vec{R}(u) = (u - u^2)\vec{i} + 2u^3\vec{j} - 3\vec{k}$,find $\int_1^2 \vec{R}(u)du$.
7. State Frenet-Serret Formulae.
8. Explain gradient and curl in general curvilinear coordinate system.
9. State Green's theorem in the plane.
10. Show that $\iint_S curl\,\vec{F}\cdot\hat{n}ds = 0$ where S is any closed surface.

SECTION-B

Answer any FIVE questions **(5 x 8 = 40)**

11. Find the equation of the tangent plane and normal line to the surface xyz = 4 at the point $\vec{i} + 2\vec{j} + 2\vec{k}$.
12. If $\phi = x^3 + y^3 + z^3 - 3xyz$, find $div\,grad\,\phi, curl\,grad\,\phi$.
13. Prove: $\nabla\times(\vec{u}\times\vec{v}) = (\nabla\cdot\vec{v})\vec{u} - (\nabla\cdot\vec{u})\vec{v} + (\vec{v}\cdot\nabla)\vec{u} - (\vec{u}\cdot\nabla)\vec{v}$.
14. Find the angle between the surfaces $x^2 - y^2 - z^2 = 11$ and $xy + yz - zx = 18$ at the point (6, 4, 3).
15. Evaluate $\int_C \vec{F}\cdot d\vec{r}$ when $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ and the curve C is the rectangle in the xy - plane bounded by $y = 0,\ x = a,\ y = b,\ x = 0$.
16. Let $\vec{F} = 2xz\vec{i} - x\vec{j} + y^2\vec{k}$ .Evaluate $\iiint_V \vec{F}dV$ where V is the region bounded by the surface $x = 0, y = 0, y = 6,\ z = x^2, z = 4$.
17. Using Green's theorem find $\int_C x^2ydx + ydy$ where C is the closed curve formed by $y^2 = x$ and $y = x$.

SECTION-C

Answer any TWO questions **(2 x 20 = 40)**

18. a) Show that $\vec{F} = (y^2 + 2xz^2)\vec{i} + (2xy - z)\vec{j} + (2x^2z - y + 2z)\vec{k}$ is irrotational and hence find its scalar potential.

    b) Show that $\nabla^2(r^n\vec{r}) = n(n+3)r^{n-2}\vec{r}$.

19. a) Evaluate $\int_C xdx + ydy$ where C is the ellipse $x^2 + 4y^2 = 4$.

    b) Evaluate $\iint_S \vec{F} \cdot \hat{n}ds$ when $\vec{F} = 18z\vec{i} - 12\vec{j} + 3y$ as S is the point of the plane $2x + 3y + 6z = 12$ which is in the first octant.

20. a) State and prove Gauss divergence theorem.

    b) Verify Stoke's theorem for $\vec{F} = (y - z + 2)\vec{i} + (yz + 4)\vec{j} - xz\vec{k}$ over the open surfaces of the cube $x = 0,\ y = 0,\ z = 0,\ x = 1,\ y = 1,\ z = 1$ not included in the xoy plane.