

STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086  
(For candidates admitted during the academic year 2011–12)

SUBJECT CODE: 11MT/MC/OD34

B. Sc. DEGREE EXAMINATION, NOVEMBER 2012  
BRANCH I - MATHEMATICS  
THIRD SEMESTER

COURSE : MAJOR – CORE  
PAPER : ORDINARY DIFFERENTIAL EQUATIONS  
TIME : 3 HOURS  
MAX. MARKS : 100

SECTION-A

Answer All the questions

(10 x 2 = 20)

1. Solve:  $a(xdy + 2ydx) = xydy$ .
2. Solve:  $p^2 - 5p + 6 = 0$  where  $p$  denotes  $\frac{dy}{dx}$ .
3. Solve:  $y = px + \frac{ap}{(1+p^2)^{\frac{1}{2}}}$  where  $p$  denotes  $\frac{dy}{dx}$ .
4. Define: orthogonal trajectories.
5. Find the particular integral of  $(D^2 + 2D + 5)y = xe^x$
6. Find the complementary function of  $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = \log x$ .
7. Solve:  $\frac{dx}{yz} = \frac{dy}{xz} = \frac{dz}{xy}$ .
8. State the differential equation of order 2 relevant to the Glucose tolerance test.
9. What is the differential equation of the motion of a rocket when the fuel is burnt off?
10. Write the differential equation for simple harmonic motion.

SECTION-B

Answer any FIVE questions

(5 x 8 = 40)

11. Solve:  $x^2 p^2 + 3xyp + 2y^2 = 0$  where  $p$  denotes  $\frac{dy}{dx}$ .
12. Solve:  $xp^2 - 2yp + x = 0$  where  $p$  denotes  $\frac{dy}{dx}$ .
13. Solve:  $\frac{d^2y}{dx^2} + y = \sec x$  using the method of variation of parameters.
14. Solve:  $\frac{dx}{-y^2 - z^2} = \frac{dy}{xy} = \frac{dz}{xz}$ .
15. Suppose a cannon ball weighing 16 pounds is shot vertically upwards with initial velocity  $v_0 = 300$  ft/s. Find (a) the velocity at any time  $t$   
(b) the maximum height attained by the cannon ball  
(ignore air resistance)
16. Derive the equation of free damped motion.
17. Explain the model for expenditure on Arms race by two countries and obtain the solution to the system.

**SECTION-C**  
**Answer any TWO questions**

(2 x 20 = 40)

18. (a) Solve:  $(y - 3x^2)dx - x(1 - xy^2)dy = 0$ .

(b) Solve the equations:

$$2 \frac{dx}{dt} + x + \frac{dy}{dt} = \cos t$$

$$\frac{dx}{dt} + 2 \frac{dy}{dt} + y = 0 \quad (8 + 12)$$

19. (a) Solve:  $(D^2 - 4D + 3)y = e^{-x} \sin x$

(b) Solve:  $3x^2 \frac{d^2 y}{dx^2} + x \frac{dy}{dx} + y = x$ . (8 + 12)

20. (a) Find the solution of the initial-value problem

$$\frac{d^2 y}{dx^2} = \frac{\omega}{T_1} \left( 1 + \left( \frac{dy}{dx} \right)^2 \right)^{\frac{1}{2}}$$

$$y(0) = \frac{T_1}{\omega}, y'(0) = 0$$

(b) Find the charge  $q(t)$  on the capacitor, and the current  $i(t)$ , for any  $t > 0$  if inductance  $L = 1$ , resistance  $R = 300$ , capacitance  $C = 5 \times 10^{-5}$  and at  $t = 0$  the switch is closed to a 40-volt battery. Also assume that  $q(0) = i(0) = 0$ . (8 + 12)

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