STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted during the academic year 2011–12 & thereafter)

SUBJECT CODE : 11MT/MC/DC14

B. Sc. DEGREE EXAMINATION, NOVEMBER 2012
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : MAJOR – CORE
PAPER : DIFFERENTIAL CALCULUS
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(10X2=20)**
ANSWER ALL THE QUESTIONS

1. Find the n[th] derivative of $e^x \sin x$.
2. Find the n[th] differential coefficient of $x^2 e^{3x}$.
3. If $u = x^3 y^4 z^2$ where $x = t^2,\ y = t^3$ and $z = t^4$, find $\frac{du}{dt}$.
4. State Euler's theorem.
5. Find the radius of curvature of the cardioid $2p^2 a = r^3$.
6. Write the formula to find the coordinates of the centre of curvature.
7. What is meant by saddle point?
8. Find the maximum product of three numbers whose sum is 39.
9. When is the curve symmetrical with respect to both axes?
10. Define a hypocycloid.

SECTION – B **(5X8=40)**
ANSWER ANY FIVE QUESTIONS

11. Find the n[th] derivative of $\frac{x^2}{(x-1)(x-2)(x-3)}$.
12. If z is a function of x and y and if $x = u - v$, $y = uv$, prove that
$(u+v)\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} + \frac{\partial z}{\partial v}$.
13. If $u = sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$. Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$.
14. Find the curvature for the curve $\sqrt{x} + \sqrt{y} = 1$ at $\left(\frac{1}{4}, \frac{1}{4}\right)$.
15. Show that the radius of curvature of the cardioid $r = a(1 + \cos\theta)$ is $\frac{2^{\frac{3}{2}}}{3}\sqrt{ar}$.

16. Find the maximum and minimum values of $2(x^2 - y^2) - x^4 + y^4$.

17. Trace the curve $r^2 = a^2 \cos 2\theta$.

SECTION – C **(2X20=40)**

ANSWER ANY TWO QUESTIONS

18. a) If $y = e^{a sin^{-1} x}$, prove that $(1 - x^2)y_2 - xy_1 - a^2 y = 0$. Hence show that $(1 - x^2)y_{n+2} - (2n + 1)xy_{n+1} - (n^2 + a^2)y_n = 0$.

b) If $u = \frac{1}{r}$ and $r^2 = (x - a)^2 + (y - b)^2 + (z - c)^2$. Prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = 0$. (10+10)

19. a) Find the $p - r$ equation of the curve $r = a\sin\theta$.

b) Show that the maximum value of $f(x, y, z) = x^2 y^2 z^2$ if $x^2 + y^2 + z^2 = a^2$ is ${a^6}/{27}$. (10+10)

20. a) Trace the curve $y^2 = x^2 \frac{(a+x)}{(b-x)}$.

b) Show that the evolute of the cycloid $x = a(\theta - \sin\theta)$ $y = a(1 - cos\,\theta)$ is another cycloid. (10+10)

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