**STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086**
**(For candidates admitted during the academic year 2011–12 & thereafter)**

**SUBJECT CODE : 11MT/MC/AT14**

**B. Sc. DEGREE EXAMINATION, NOVEMBER 2012**
**BRANCH I - MATHEMATICS**
**FIRST SEMESTER**

**COURSE : MAJOR – CORE**
**PAPER : ALGEBRA AND TRIGONOMETRY**
**TIME : 3 HOURS** **MAX. MARKS : 100**

**SECTION – A** **(10X2=20)**
**ANSWER ALL THE QUESTIONS**

1. Solve the equation $x^3 + 6x + 20 = 0$, one root being $1 + 3i$.
2. Multiply the roots of $x^3 - 3x + 1 = 0$ by $10$.
3. Define reciprocal equation.
4. Find the number of real roots of the equation $x^3 + 18x - 6 = 0$.
5. State Cayley Hamilton theorem.
6. Define similar matrices.
7. Express $\cos 5\theta$ interms of $\cos\theta$.
8. Prove that $\sin h2x = 2\sin hx \cos hx$.
9. Separate into real and imaginary parts of $\cos(x + iy)$.
10. Find $Log(-2)$.

**SECTION – B** **(5X8=40)**
**ANSWER ANY FIVE QUESTIONS**

11. Find the sum of the cubes of the roots of $x^3 - 7x + 6 = 0$.
12. Show that the equation $x^4 - 3x^3 + 4x^2 - 2x + 1 = 0$ can be transformed into a reciprocal equation by diminishing the roots by unity. Hence solve the equation.
13. Test for consistency and hence solve $x - 2y + 3z = 2,\ 2x - 3z = 3, x + y + z = 0$.
14. Find the rank of the matrix $\begin{pmatrix} 4 & -5 & 1 & 2 \\ 3 & 1 & -2 & 9 \\ 1 & 4 & 1 & 5 \end{pmatrix}$.
15. Prove that $cos^4\theta sin^3\theta = \frac{-1}{2^6}[\sin 7\theta + \sin\ 5\theta - 3\sin 3\theta - 3\sin\theta]$.
16. If $\tan(A + iB) = x + iy$ prove that $x^2 + y^2 + 2x\,cot\,2A = 1$ and $x^2 + y^2 - 2y\cot\ h2B + 1 = 0$.
17. Separate the real and imaginary parts of $(\alpha + i\beta)^{x+iy}$.

**SECTION – C** **(2X20=40)**

**ANSWER ANY TWO QUESTIONS**

18. (i) If $\alpha, \beta, \gamma$ are the roots of the equation $x^3 + px^2 + qx + r = 0$, form the equation whose roots are $\alpha - \frac{1}{\beta\gamma}, \beta - \frac{1}{\gamma\alpha}, \gamma - \frac{1}{\alpha\beta}$.

(ii) Solve $x^5 - 5x^4 + 9x^3 - 9x^2 + 5x - 1 = 0$. (10+10)

19. (i) Find the eigen values and eigen vectors of $\begin{pmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{pmatrix}$.

(ii) Verify Cayley-Hamilton theorem and hence find inverse for $\begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$.

(10+10)

20. (i) Prove that $\tan h^{-1} x = \frac{1}{2} \log\left(\frac{1+x}{1-x}\right)$.

(ii) Evaluate $\lim_{\theta \to 0} \frac{\tan\theta - \sin\theta}{\theta^3}$.

(iii) Show that if $\lambda$ is an eigen value of the matrix $A$, then $\lambda^n$ is an eigen value of $A^n$, where $n$ is a positive integer.

(7+6+7)