STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 86
(For candidates admitted from the academic year 2023 – 2024)

M.Sc. DEGREE EXAMINATION, APRIL 2024
BRANCH I - MATHEMATICS
SECOND SEMESTER

COURSE : ELECTIVE
PAPER : MECHANICS
SUBJECT CODE : 23MT/PE/ME15
TIME : 3 HOURS **MAX. MARKS: 100**

Q. No.	SECTION A ($5 \times 2 = 10$) **Answer ALL questions**	CO	KL
1.	Write Lagrange's equation in Nielsen form.	1	1
2.	State the Hamiton's principle for monogenic system.	1	1
3.	Define instantaneous axis of rotation.	1	1
4.	State Hertz's principle of least curvature.	1	1
5.	Define configuration phase and phase space.	1	1

Q. No.	SECTION B ($10 \times 1 = 10$) **Answer ALL questions**	CO	KL
6.	A reference frame in which the equation $F = \frac{dp}{dt}$ is valid is termed: a) Inertial system b) Curvilinear system c) Non-inertial system d) Rotating system	2	2
7.	What is the degree of freedom of a system consisting of 2N particles, free from constraints? a) 2N b) 3N c) 4N d) 6N	2	2
8.	What method is used to eliminate extra virtual displacements? a) Hamilton's principle b) Poisson's equation c) Lagrange multipliers d) Newton's second law	2	2
9.	Which curve was discovered to be the solution to the Brachistochrone problem? a) Circle b) Parabola c) Cycloid d) Ellipse	2	2
10.	Which axis does the Pitch rotation occur around in the Tait-Bryan angles representation? a) X-axis b) Y-axis c) Z-axis d) None of the above	2	2

11.	What equation represents the relationship between the angular momentum vector (**L**) and the angular velocity vector (**ω**) for a rigid body? a) $L = I \times \omega$ b) $L = I \pm \omega$ c) $L = I \cdot \omega$ d) $L = \frac{dI}{dt}$	2	2
12.	If the Lagrangian of a system is not explicitly dependent on time, then: a) The system does not conserve energy b) The system has no conserved quantities c) Cyclic coordinates are linear functions of time in steady motion d) The system undergoes chaotic motion	2	2
13.	Which characteristic of a system's transfer function can be determined using Routh's criterion? a) Gain margin b) Phase margin c) Number of poles d) Stability	2	2
14.	What is the role of a generating function of a canonical transformation? a) It determines the form of Hamilton's equations. b) It serves as a bridge between two sets of canonical variables. c) It represents the total energy of the system. d) It determines the potential energy of the system.	2	2
15.	Are Poisson brackets invariant under canonical transformations? a) Yes, they remain unchanged b) No, they change arbitrarily c) Only in certain cases d) It depends on the specific transformation	2	2

Q. No.	**SECTION C ($2 \times 15 = 30$)** **Answer ANY TWO questions**	**CO**	**KL**
16.	(a) Discuss the problem of Atwood's machine using Lagrange's formulation. (b) Discuss the motion of one particle using Cartesian coordinates.	3	3
17.	(a) Derive the Lagrange's equation from Hamilton's Principle. (b) Demonstrate the reciprocal relationship between the Poisson bracket and the Lagrange bracket.	3	3

18.	(a) Obtain the total energy for a system consisting of a heavy symmetrical top with one fixed point. (b) Obtain Hamilton's canonical equations of motion.	3	3
19.	State and Prove the principle of least action in Jacobi form	3	3

Q. No.	**SECTION D ($2 \times 15 = 30$)** **Answer ANY TWO questions**	**CO**	**KL**
20.	Prove that the magnitude R of the position vector for the centre of mass from an arbitrary origin is given by the equation $M^2R^2 = M\sum_i m_i r_i^2 - \frac{1}{2}\sum_{i,j} m_i m_j r_{ij}^2$.	4	4
21.	(a) Illustrate the application of the Lagrange multiplier method by considering the example of a loop rolling without slipping down an inclined plane. (b) Obtain the shortest distance between two points in a plane.	4	4
22.	(a) The Lagrangian for system can be written as $L = a\dot{x}^2 + b\frac{\dot{y}}{x} + c\,\dot{x}\dot{y} + fy^2\dot{x}\dot{z} + g\dot{y}^2 - k\sqrt{x^2+y^2}$, where a, b, c, f, g and k are constants. What is the Hamiltonian? What quantities are conserved? (b) Derive the expression for Coriolis force.	4	4
23.	Solve the problem of the simple harmonic oscillator in one dimension using a canonical transformation.	4	4

Q. No.	**SECTION E ($2 \times 10 = 20$)** **Answer ANY TWO questions**	**CO**	**KL**
24.	State and prove D'Alembert's principle.	5	5
25.	(a) Obtain the minimum surface of revolution (b) Show directly that the transformation $Q = \log\left(\frac{1}{q}\sin p\right)$, $P = q\cot p$ is canonical.	5	5
26.	Prove that the real orthogonal matrix specifying the physical motion of a rigid body with one point fixed always has the eigen value +1.	5	5
27.	Derive the Hamiton's Equations from variational principle.	5	5