STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 86
(For candidates admitted from the academic year 2023 – 2024 and thereafter)

M.Sc. DEGREE EXAMINATION, APRIL 2024
BRANCH I - MATHEMATICS
SECOND SEMESTER

COURSE : MAJOR CORE
PAPER : TOPOLOGY
SUBJECT CODE : 23MT/PC/TO24
TIME : 3 HOURS **MAX. MARKS: 100**

Q. No.	SECTION A $(5 \times 2 = 10)$ Answer ALL questions	CO	KL
1.	Define a topological space with an example.	1	1
2.	Which subsets in an indiscrete topology on a set X are open?	1	1
3.	What do you mean by a subspace topology?	1	1
4.	Define Hausdorff space.	1	1
5.	Explain a limit point compact space briefly.	1	1

Q. No.	SECTION B$(10 \times 1 = 10)$. Answer ALL questions	CO	KL
6.	Which of the following is an open set in R? (i) [1,2] (ii) (1,2] (iii) [1,2) (iv) (1,2)	2	2
7.	Y= [-1, 1], a subspace of R. Which of the following is both open in Y and R? (i) $\left\{x \in R/\frac{1}{2} < \lvert x\rvert < 1\right\}$ (ii) $\left\{x \in R/\frac{1}{2} < \lvert x\rvert \leq 1\right\}$ (iii) $\left\{x \in R/\frac{1}{2} \leq \lvert x\rvert < 1\right\}$ (iv) $\left\{x \in R/\frac{1}{2} \leq \lvert x\rvert \leq 1\right\}$	2	2
8.	Which of the following is not true in a topological space X? (i) $\emptyset$ and X are open (ii) $\emptyset$ and X are closed (iii) arbitrary intersections open sets are open (iv) arbitrary unions of open sets are open	2	2
9.	Which of the following is true? (i) (0,1]is compact (ii) (0,1]is closed (iii) (0,1]is not bounded (iv) [0,1] is compact	2	2

10.	In a discrete topology on a set X (i) all subsets of X are open (ii) all one point subsets of X alone are closed (iii) all one point subsets of X alone are open (iv) only X and $\emptyset$ are open	2	2
11.	If every point of a non-empty compact Hausdorff space X is a limit point of X then X is ----------. (i) countable (ii) uncountable (iii) bounded (iv) unbounded	2	2
12.	Any ---------- Hausdorff space is normal. (i) closed (ii) open (iii) bounded (iv) compact	2	2
13.	Which of the following spaces is always normal? (i) Hausdorff space (ii) compact space (iii) metric space (iv) topological space	2	2
14.	A subspace of a regular space ----------. (i) need not be regular (ii) normal (iii) regular (iv) compact	2	2
15.	Which of the following statement is true? (i) Every metric space is normal (ii) Every compact metric space is normal (iii) Every normal space is compact (iv) Every compact metric space is metrizable	2	2

Q. No.	SECTION C $(2 \times 15 = 30)$ Answer ANY TWO questions	CO	KL
16.	Can we say that if $f: X \rightarrow Y$ is a function, X and Y are topological spaces, the following statements are equivalent? a) f is continuous b) for every subset A of X, $f(\bar{A}) \subset \overline{f(A)}$ c) for every closed set B in Y, $f^{-1}(B)$ is closed in X. Prove the supporting results.	3	3
17.	How would you say that any interval or a ray in a linear continuum L in the order topology is connected?	3	3

18.	Let X be a metrizable space. Then prove that the following are equivalent: a) X is compact. b) X is limit point compact. c) X is sequentially compact.	3	3
19.	State and prove the Tietz extension theorem.	3	3

Q. No.	**SECTION D $(2 \times 15 = 30)$** **Answer ANY TWO questions**	**CO**	**KL**
20.	Examine the following statement is true: "If $f: X \to Y$ is a bijective continuous function, X is compact and Y is Hausdorff, then f is a homeomorphism".	4	4
21.	Is the union of connected subspaces of X having a point in common is connected? Justify it.	4	4
22.	Prove that the product of finitely compact spaces is compact? Justify it with a supporting proof.	4	4
23.	Demonstrate Urysohn lemma.	4	4

Q. No.	**SECTION E $(2 \times 10 = 20)$** **Answer ANY TWO questions**	**CO**	**KL**
24.	Defend the topologies induced by the euclidean metric and the square metric are the same as the product topology on $\mathbb{R}^n$.	5	5
25.	Prove that a subspace A of $\mathbb{R}^n$ is compact if and only if it is closed and bounded in the Euclidean metric d or the square metric ρ.	5	5
26.	Is every closed interval in $\mathbb{R}$ uncountable? Justify it.	5	5
27.	State Urysohn metrization theorem and investigate it by giving a supportive proof.	5	5

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