STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 86
(For candidates admitted from the academic year 2023 – 2024)

M.Sc. DEGREE EXAMINATION, APRIL 2024
BRANCH I - MATHEMATICS
SECOND SEMESTER

COURSE	**:**	**MAJOR CORE**	
PAPER	**:**	**LINEAR ALGEBRA**	
SUBJECT CODE	**:**	**23MT/PC/LA24**	
TIME	**:**	**3 HOURS**	**MAX. MARKS: 100**

Q. No.	SECTION A $(5 \times 2 = 10)$ Answer ALL questions	CO	KL
1.	What are the characteristic values of a Nilpotent transformation on a finite dimensional vector space V over a field F?	1	1
2.	If V is a vector space over a field F and $T \in A(V)$, then how would you convert V into an $F[x]$- module?	1	1
3.	What is the relationship between the characteristic and minimal polynomials for a linear operator T on a finite dimensional vector space?	1	1
4.	When do you say that the complex $n \times n$ matrices A and B are unitarily equivalent?	1	1
5.	State Principal Axis Theorem.	1	1

Q. No.	SECTION B $(10 \times 1 = 10)$ Answer ALL questions	CO	KL
6.	If T $\in A(V)$, then which of the following is an example of an invariant subspace? (i) $Ker\ T$ (ii) VT (iii) $\{0\}$ (iv) All of the above	2	2
7.	The index of nilpotence of a nilpotent transformation $T: \mathbb{R}^{(2)} \to \mathbb{R}^{(2)}$ defined by $(x, y)T = (x - y, x - y)$ is ____________ (i) 3 (ii) 2 (iii) 4 (iv) 5	2	2
8.	The Jordan form of the matrix $\begin{pmatrix} 1 & 0 \\ -3 & 5 \end{pmatrix}$ is ____________ (i) $\begin{pmatrix} 1 & 0 \\ 0 & 5 \end{pmatrix}$ (ii) $\begin{pmatrix} 1 & 1 \\ 0 & 5 \end{pmatrix}$ (iii) $\begin{pmatrix} 1 & 0 \\ 3 & 5 \end{pmatrix}$ (iv) $\begin{pmatrix} 1 & 1 \\ -3 & 5 \end{pmatrix}$	2	2

9.	The companion matrix of the polynomial $x^3 + 7x^2 - 9x + 3$ is ________ (i) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 7 & -9 & 3 \end{pmatrix}$ (ii) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 3 & -9 & 7 \end{pmatrix}$ (iii) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -7 & 9 & -3 \end{pmatrix}$ (iv) $\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & 9 & -7 \end{pmatrix}$	2	2
10.	The minimal and the characteristic polynomial of an $n \times n$ identity matrix are respectively _____________ (i) $x - 1$ and $x^n - 1$ (ii) $x^n - 1$ and $(x-1)^n$ (iii) $x - 1$ and $(x-1)^n$ (iv) $(x-1)^n$ and $x^n - 1$	2	2
11.	The characteristic values of an $n \times n$ triangular matrix are always (i) Non-negative entries of the main diagonal (ii) The super diagonal entries (iii) The main diagonal entries (iv) Zeros	2	2
12.	The characteristic values of a self-adjoint linear operator on a finite dimensional inner product space are (i) Only zeros (ii) Complex numbers (iii) Real numbers (iv) Of absolute value 1 only	2	2
13.	A complex $n \times n$ matrix A is called unitary if (i) A = A* (ii) AA* = A*A (iii) A = - A* (iv) AA* = I = A*A	2	2
14.	Every entry on the main diagonal of a positive matrix is ------ (i) Positive (ii) Zero (iii) Negative (iv) All of the above	2	2
15.	A form f on a finite dimensional real or complex inner product space V is positive if _____________ (i) f is Hermitian and $f(\alpha, \alpha) > 0$ for all $\alpha \neq 0$ in V (ii) f is Hermitian and $f(\alpha, \alpha) \geq 0$ for all $\alpha \neq 0$ in V (iii) f is Hermitian and $f(\alpha, \alpha) > 0$ for every α in V (iv) f is Hermitian and $f(\alpha, \alpha) \geq 0$ for every α in V	2	2

Q. No.	**SECTION C** $(\mathbf{2 \times 15 = 30})$ **Answer ANY TWO questions**	**CO**	**KL**
16.	Apply your understanding of the relation between a linear transformation and a matrix in proving the following: If $T \in A(V)$ has all its characteristic roots in a field F, then there is a basis of V in which the matrix of T is triangular.	3	3
17.	How can you apply the fact that $T \in A(V)$ has minimal polynomial $p(x)$ in proving that V can be decomposed as $V = V_1 \oplus V_2 \oplus \dots \oplus V_k$ where each V_i is invariant subspace of V under T and the minimal polynomial of the induced transformation T_i is a factor of $p(x)$	3	3
18.	Let V be a finite-dimensional vector space over the field F and let T be a linear operator on V. Then prove that T is triangulable if and only if the minimal polynomial for T is a product of linear polynomials over F.	3	3
19.	Let V and W be finite-dimensional inner product spaces over the same field, having the same dimension. If T is a linear transformation from V into W, then show that the following are equivalent. (i) T preserves inner products. (ii) T is an (inner product space) isomorphism. (iii) T carries every orthonormal basis for V onto an orthonormal basis for W. (iv) T carries some orthonormal basis for V onto an orthonormal basis for W.	3	3

Q. No.	**SECTION D** $(\mathbf{2 \times 15 = 30})$ **Answer ANY TWO questions**	**CO**	**KL**
20.	Show that the invariants of a nilpotent transformation T are unique.	4	4
21.	Show that every linear transformation T $\in A(V)$ satisfies its characteristic polynomial.	4	4
22.	If T is a linear operator on a finite dimensional inner product space V and A is the matrix of T in the ordered orthonormal basis $B = \{\alpha_1, \alpha_2, \dots, \alpha_n\}$ of V, then prove that the matrix of T* is the conjugate transpose of the matrix of T.	4	4
23.	How would you relate a form and a linear operator to have an isomorphism between L(V,V) and the space of all forms?	4	4

Q. No.	**SECTION E** $(\mathbf{2 \times 10 = 20})$ **Answer ANY TWO questions**	CO	KL
24.	Calculate all possible elementary divisors and rational canonical forms for the 6×6 matrices having $(x-1)(x^2+1)^2$ as minimal polynomial.	5	5
25.	Find the Characteristic and minimal polynomial of $\begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{pmatrix}$	5	5
26.	When will $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be (i) Orthogonal (ii) Unitary? Justify your answer.	5	5
27.	Let F = $\mathbb{R}$ or $\mathbb{C}$ and let A be an $n \times n$ matrix over F. Then prove that the function g defined by g (X, Y) = Y*AX is a positive form on the space $F^{n\times 1}$ if and only if there is an invertible $n \times n$ matrix over F such that A = P*P	5	5