STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the year 2019-20 and thereafter)

M. Sc. DEGREE EXAMINATION, APRIL 2024
BRANCH I - MATHEMATICS
FOURTH SEMESTER

COURSE	**:**	**ELECTIVE**
PAPER	**:**	**MATHEMATICAL STATISTICS**
SUBJECT CODE	**:**	**19MT/PE/MS15**
TIME	**:**	**3 HOURS** **MAX. MARKS : 100**

SECTION – A
ANSWER ALL THE QUESTIONS (5 × 2 = 10)

1. Define characteristic function of a random variable X.
2. Give the density function of a random variate following Gamma distribution.
3. Define stochastic convergence of a sequence of random variables.
4. What is a simple random sample?
5. Give an example of the unbiased estimate of mean of a normal population.

SECTION – B
ANSWER ANY FIVE QUESTIONS (5 × 6 = 30)

6. Find the characteristic function of a random variable X that has a Poisson distribution.
7. Prove that the sum of two independent random variables with Gamma distribution is again a random variate having gamma distribution.
8. Define beta distribution and find the first two moments of the Beta distribution.
9. State and prove Bernoulli's law of large numbers.
10. A coin is thrown 100 times. The number 1 is assigned to the appearance of head and 0 to the appearance of tail. The probability of each of these events is equal to 0 .5. What is the probability that heads will appear more than 50 times and less than 60 times?
11. Calculate the density function of the arithmetic mean of an independent normally distributed random variable.
12. Find a sufficient estimator for the mean and variance of a random sample of size *n* drawn from a normal population.

SECTION – C
ANSWER ANY THREE QUESTIONS (3 × 20 = 60)

13. Show that we can determine the distribution function uniquely from the characteristic function by proving the Levy's theorem.

14. Derive the characteristic function of the Cauchy distribution from its density function and hence show that none of the moments of the Cauchy distribution exist.

15. State and prove the following results
(i) Poisson's law of large numbers,
(ii) Chebychev's law of large numbers,
(iii) Kinchin's law of large numbers.

16. Derive the density function of a Chi Square variant with (n – 1) degrees of freedom.

17. State and prove the Rao Cramer inequality. Also find the conditions for the equality sign in the Rao Cramer inequality.

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