STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2019-20 & thereafter)

M. Sc. DEGREE EXAMINATION, APRIL 2024
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE	**: ELECTIVE**	
PAPER	**: CALCULUS OF VARIATION AND INTEGRAL EQUATIONS**	
SUBJECT CODE	**: 19MT/PE/CI15**	
TIME	**: 3 hours**	**MAX. MARKS: 100**

Section – A
Answer ALL questions (5 × 2 = 10)

1. Write the Euler's equation for an extremizing the functional $I[y(x)] = \int_a^b F(x, y(x), y'(x))dx$ if F depends only on x and y'.
2. State the Weirstrass – Erdmann corner conditions.
3. What is seperable kernel?
4. Define Fredholm alternative.
5. List out the application of Green's functions.

Section – B
Answer any FIVE questions (5 × 6 = 30)

6. Show that there exists an extremal through any two points with distinct abscissae for the functional $I = \int_0^b e^{-2y^2}(y'^2 - 1)dx$
7. Find the extremum of the functional $I = \int_{x_1}^{x_2}(y'^2 + z'^2 + 2yz)\, dx$ with $y(0) = 0$.
8. State Brachistochrone problem and also solve it.
9. Describe briefly the convolution integral.
10. Solve the Fredholm integral equation of the second kind $g(s) = s + \lambda\int_0^1 (st^2 + s^2t)g(t)dt$.
11. Obtain the resolvent kernel for $g(s) = f(s) + \lambda\int_0^s e^{s-t}g(t)dt$.
12. Explain Dirac delta function.

Section – C
Answer any THREE question (3 × 20 = 60)

13. (a) Determine the shape of a solid of revolution moving in a flow of gas with least resistance.

    (b) Derive Euler-Poisson equation in testing the extremum of the functional depending on higher-order derivatives.

14. (a) Derive the transversality condition to find the extremal for functional with moving end points.

    (b) Find the shortest path from the point $A(-2,3)$ to the point $B(2,3)$ located in the region $y \le x^2$.

15. (a) Find the Eigen values and Eigen function of the homogeneous integral equation.

    $$g(s) = \lambda \int_1^2 \left[st + \frac{1}{st} \right] g(t)dt .$$

    (b) Solve the integral equation $g(s) = 1 + \lambda \int_0^{\pi} sin(s + t)g(t)dt.$

16. (a) Solve the inhomogeneous Fredholm integral equation of the second kind $g(s) = 2s + \lambda \int_0^1 (s + t)g(t)dt$ by the method of successive approximation to the third order.

    (b) Explain briefly Fredholm theorem.

17. Reduce the boundary value problem to a Fredholm integral equation

    $y'' + \lambda y = 0,\ 0 \le s \le 1,\ \ y(0) = 0,\ \ y'(1) + v_2 y(1) = 1.$

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