STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019 – 20 and thereafter)

SUBJECT CODE : 19MT/PC/PS34

M. Sc. DEGREE EXAMINATION, NOVEMBER 2023
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : CORE
PAPER : PROBABILITY AND STOCHASTIC PROCESSES
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A

ANSWER ALL THE QUESTIONS: **(5 × 2 = 10)**

1. Define conditional probability density functions.
2. What is meant by “Infinite server Poisson queue”?
3. Define branching process.
4. Give an example of continuous Markov-chain.
5. State Martingale stopping theorem.

SECTION – B

ANSWER ANY FIVE QUESTIONS: **(5 × 6 = 30)**

6. State and prove Borel-Cantelli lemma.
7. In an election, candidate A receives n votes and candidate B receives m votes, where $n > m$. By assuming all orderings are equally likely, show that the probability that A is always ahead in the count of votes is $\frac{n-m}{n+m}$.
8. If $N_i(t)$ represents the number of type i events that occur by time $t, i = 1,2$, then prove that $N_1(t)$ and $N_2(t)$ are independent Poisson random variables having respective means λtp and $\lambda t(1-p)$, where $p = \frac{1}{t}\int_0^t P(s)ds$.
9. If j is recurrent, then prove that the set of probabilities $\{f_{ij}, i \in T\}$ satisfies $f_{ij} = \sum_{k\in T} P_{ik} f_{kj} + \sum_{k\in R} P_{ik}, i \in T$, where R denotes the set of states communicating with j.
10. Explain Birth and Death processes with suitable examples.
11. Consider a Yule process with $X(0) = 1$. Compute the expected sum of the ages of the members of the population at time t.
12. Let X be such that $E(X) = 0$ and $P\{-\alpha \le X \le \beta\} = 1$. Then for any convex function f, prove that $E[f(X)] \le \frac{\beta}{\alpha+\beta} f(-\alpha) + \frac{\alpha}{\alpha+\beta} f(\beta)$.

SECTION – C

ANSWER ANY THREE QUESTIONS: **(3 × 20 = 60)**

13. (a) If $\{E_n, n \geq 1\}$ is either an increasing or decreasing sequence of events, then prove that $\lim_{n\to\infty} P(E_n) = P\left(\lim_{n\to\infty} E_n\right)$.

(b) Find the mean and variance of the matching problem. (10+10)

14. Write down the two definitions of the Poisson process and demonstrate that they are equivalent.

15. (a) State and prove Chapman-Kolmogorov equations.

(b) Consider the gambler's ruin problem with $p = 4$ and $n = 6$. Starting in state 3, determine

(i) the expected amount of time spent in state 3.

(ii) the expected number of visits to state 2.

(iii) the probability of ever visiting state 4. (10+10)

16. (a) State and prove Kolmogorov backward equations.

(b) Explain simple epidemic model of a pure birth process. (10+10)

17. (a) If N is a random time for the Martingale $\{Z_n\}$, then prove that the stopped process $\{\overline{Z_n}\}$ is also a Martingale.

(b) State and prove Wald's equation. (12+8)

⅄⅄⅄⅄⅄⅄⅄⅄⅄⅄