STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI-600 086
(For candidates admitted during the academic year 2019 – 20 & thereafter)

M.Sc. DEGREE EXAMINATION, APRIL 2024
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE	**:**	**CORE**	
PAPER	**:**	**DIFFERENTIAL GEOMETRY**	
SUBJECT CODE	**:**	**19MT/PC/DG44**	
TIME	**:**	**3 HOURS**	**MAX: 100 MARKS**

SECTION – A
Answer all the questions (5 × 2 = 10)

1. Define arc length of a curve.
2. When a surface is said to be orientable?
3. Show that $\|\sigma_u \times \sigma_v\| = (EG - F^2)^{\frac{1}{2}}$.
4. Find the second fundamental form of a plane $\sigma(u, v) = a + up + vq$.
5. Define gaussian and mean curvatures.

SECTION – B
Answer any five questions (5 × 6 = 30)

6. Let $\gamma(s)$ be a unit-speed curve and let $\varphi(s)$ be the angle through which a fixed unit vector must be rotated anti-clockwise to bring it into coincidence with the unit tangent vector t of γ. Then prove that $\kappa_s = \frac{d\varphi}{ds}$.
7. Compute the arc length of the curve $\gamma(t) = (e^{kt}cost, e^{kt}sint)$, where $-\infty < t < \infty$ and k is a non-zero constant.
8. Find the equation of the tangent plane of the surface patch $\sigma(u, v) = (u, v, u^2 - v^2)$ at (1,1,0).
9. Prove that any tangent developable is isometric to (part of) a plane.
10. Calculate the first fundamental form of a sphere $\sigma(\theta, \varphi) = (cos\theta\ cos\varphi, cos\theta\ sin\varphi, sin\theta)$.
11. State and prove Meusiner's theorem.
12. Define geodesic and prove that any geodesic has constant speed.

SECTION – C

Answer any three questions (3 × 20 = 60)

13. Let $\gamma(t)$ be a regular curve in R^3. Then prove that its curvature is $\kappa = \frac{\|\ddot{\gamma}\times\dot{\gamma}\|}{\|\dot{\gamma}\|^3}$ and its torsion is $\tau = \frac{(\dot{\gamma}\times\ddot{\gamma})\cdot\dddot{\gamma}}{\|\dot{\gamma}\times\ddot{\gamma}\|^2}$.

14. (a) Let $f: S_1 \to S_2$ be a diffeomorphism. If σ_1 is an allowable surface patch on S_1 then prove that $f \circ \sigma_1$ is an allowable surface patch on S_2.
    (b) Describe an atlas for a generalized cylinder. (10+10)

15. Prove that a diffeomorphism $f: S_1 \to S_2$ is isometry if and only if, for any surface patch σ_1 of S_1, the patches σ_1 and $f \circ \sigma_1$ of S_1 and S_2 respectively, have the same first fundamental form.

16. (a) State and prove Euler's theorem.
    (b) Let N be the standard unit normal of a surface patch $\sigma(u, v)$. Then prove that $N_u = a\sigma_u + b\sigma_v$ and $N_v = c\sigma_u + d\sigma_v$ where $\begin{pmatrix} a & c \\ b & d \end{pmatrix} = -\mathcal{F}_I^{-1}\mathcal{F}_{II}$. (12+8)

17. Calculate the gaussian curvature of the helicoid $\sigma(u, v) = (v\ cosu, v\ sinu, \lambda u)$and catenoid $\sigma(u, v) = (coshu\ cosv,\ coshu\ sinv, u)$.

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