STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI – 600 086
(For candidates admitted during the academic year 2019 – 20 & thereafter)

M.Sc. DEGREE EXAMINATION, April 2024
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE	**: CORE**
PAPER	**: CONTINUUM AND FLUID MECHANICS**
SUBJECT CODE	**: 19MT/PC/CF44**
TIME	**: 3 HOURS** **MAX. MARKS : 100**

Section – A
Answer ALL questions $(\mathbf{5 \times 2 = 10})$

1. Define summation convention with an example.
2. Define homogeneous and anisotropic materials.
3. Define vorticity vector and vortex line.
4. Draw a venturi tube and list its applications.
5. State uniqueness theorem.

Section – B
Answer ANY FIVE questions $(\mathbf{5 \times 6 = 30})$

6. Let *a* and *b* be arbitrary vectors. Show that the second and third principal invariants of $a \otimes b$ are zero. If *a* and *b* are orthogonal, show that the first principal invariant of $a \otimes b$ is also zero.
7. The stress tensor at point P is given by the array $\sum = \begin{pmatrix} 7 & -5 & 0 \\ -5 & 3 & 1 \\ 0 & 1 & 2 \end{pmatrix}$ Determine the stress vector on the plane passing through P and parallel to the plane ABC, given that $PA = 4''$, $PB = 2''$ and $PC = 6''$.
8. Discuss the Doublet in uniform stream.
9. Derive the Bernoulli's equation for the homogeneous and incompressible fluid.
10. Solve the steady flow problem through a channel of uniform rectangular cross- section.
11. Derive the acceleration of a fluid.
12. A continuum body undergoes the displacement. $u = (3X_2 - 4X_3)\vec{e_1} + (2X_1 - X_3)\vec{e_2} + (4X_2 - X_1)\vec{e_3}$. Determine the displaced position of the vector joining particles A(1,0,3) and B(3,6,6), assuming superposed material and spatial axes.

Section – C
Answer ANY THREE questions $(\mathbf{3 \times 20 = 60})$

13. (a) Prove the following identities

(i) $Ia = a$ (ii) $(a \otimes b)c = (b \cdot c)a$ (iii) $(a \otimes b - b \otimes a)c = c \times (a \times b)$

(iv) $A(a \otimes b) = (Aa) \otimes b$ (v) $(a \otimes b)(c \otimes d) = (b \cdot c)(a \otimes d)$.

(b) Show that $a_{ij} = a_{ji}$ if and only if $\varepsilon_{ijk} a_{jk} = 0$.

14. (a) Determine the Cauchy stress quadric stress at P for the following states of stress:

(i) Uniform tension $\sigma_{11} = \sigma_{22} = \sigma_{33} = \sigma$; $\sigma_{12} = \sigma_{13} = \sigma_{23} = 0$.

(ii) Uniaxial tension $\sigma_{11} = \sigma$; $\sigma_{22} = \sigma_{33} =$ $\sigma_{12} = \sigma_{13} = \sigma_{23} = 0$.

(iii) Simple shear $\sigma_{12} = \sigma_{21} = \tau$; $\sigma_{11} = \sigma_{22} = \sigma_{33} = \sigma_{13} = \sigma_{23} = 0$.

(iv) Plane stress with $\sigma_{11} = \sigma_{22} = \sigma$; $\sigma_{12} = \sigma_{21} = \tau; \sigma_{33} = \sigma_{31} = \sigma_{23} = 0$.

(b) With respect to superposed material axes X_i and spatial axes x_i, the displacement field of a continuum body is given by $x_1 = X_1,\ x_2 = X_2 + AX_3,\ x_3 = X_3 + AX_2$, where A is a constant. Determine the displaced location of the material particles which originally comprise (i) the plane circular surface $X_1 = 0,\ X_2^2 + X_3^2 = 1/(1 - A^2)$, (ii) the infinitesimal cube with edges along the coordinate axes of length $dX_i = dX$. Sketch the displaced configuration for (i) and (ii) if A=1/2.

15. (a) Derive the steady state continuity equation for an incompressible fluid.

(b) At any point in an incompressible fluid having spherical polar coordinates (r, θ, ψ), the velocity components are $\left[2\sqrt{K}\, r^{-3} \cos\theta,\ \sqrt{K}\, r^{-3} \sin\theta, 0\right]$, where K is a constant. Show that the velocity is of the potential kind. Find the velocity potential and the equations of streamlines.

16. (a) Prove that at any point P of a moving inviscid fluid the pressure p is the same in all directions.

(b) Describe the irrotational motion of an incompressible liquid for which $w = ik \log z$.

17. (a) Discuss the Poiseuille flow through the tube of uniform circular cross section and obtain the volume of fluid discharged over any section per unit time.

(b) Derive the Navier- Stokes equation of motion of a viscous fluid.

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