STELLA MARIS COLLEGE (AUTONOMOUS), CHENNAI-600 086
(For candidates admitted during the academic year 2019 – 20 & thereafter)

M.Sc. DEGREE EXAMINATION, April 2024
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE	**:**	**CORE**	
PAPER	**:**	**COMPLEX ANALYSIS**	
SUBJECT CODE	**:**	**19MT/PC/CA44**	
TIME	**:**	**3 HOURS**	**MAX: 100 MARKS**

SECTION – A
Answer ALL the questions $(5 \times 2 = 10)$

1. Define index of a point
2. Define simple connectivity with suitable example
3. Prove that $\prod_{n=2}^{\infty}(1 - \frac{1}{n^2}) = \frac{1}{2}$
4. Define Normal Family of functions
5. Define Free Boundary arc of a region

SECTION – B
Answer ANY FIVE questions $(5 \times 6 = 30)$

6. Evaluate $\int_{|z|=\rho} \frac{|dz|}{|z-a|^2}$, under the condition $|a| \neq \rho$.
7. Suppose that $u(z)$ is harmonic for $|z| < R$, continuous for $|z| \leq R$. Then prove that $u(a) = \frac{1}{2\pi}\int_{|z|=R} \frac{R^2-|a|^2}{|z-a|^2} u(z)d\theta$ for all $|a| < R$.
8. Prove that $\Gamma\,(1 - z)\Gamma\,(z) = \frac{\pi}{sin\pi z}$.
9. If S is complete then $\mathfrak{J}$ is normal if and only if it is totally bounded.
10. State and prove Reflection Principle.
11. Define Harmonic Function and state any two properties of Harmonic function.
12. A family $\mathfrak{J}$ is normal if and only if its closure $\mathfrak{J}^{-}$ with respect to the distance function $\rho(f,g) = \sum_{k=1}^{\infty} \delta_k(f,g)2^{-k}$ is compact.

SECTION – C
Answer ANY THREE questions $(3 \times 20 = 60)$

13. (a) State and prove Cauchy's theorem in a disk.
    (b) State and prove Cauchy Integral Formula.
    (10 + 10)
14. (a) State and prove Schwarz's theorem.
    (b) State and prove mean value property of Harmonic functions
    (10 + 10)
15. (a) Prove that the function equation $\zeta(s) = 2^s\pi^{s-1}\sin\frac{\pi s}{2}\Gamma\,(1 - s)\zeta(1 - s)$.
    (b) For $\sigma = Re\,s > 1$ show that $1/\zeta(s) = \prod_{n=1}^{\infty}(1 - p_n^{-s})$ where $p_1, p_2, ...$ is the ascending sequence of primes.
    (10 + 10)

16. State and prove Riemann Mapping Theorem.

17. State and prove Schwarz Christoffel formula.

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