STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 86
(For candidates admitted from the academic year 2023 – 2024)

B.Sc. DEGREE EXAMINATION, APRIL 2024
BRANCH I - MATHEMATICS
SECOND SEMESTER

COURSE : MAJOR CORE
PAPER : INTEGRAL CALCULUS
SUBJECT CODE : 23MT/MC/IC23
TIME : 3 HOURS **MAX. MARKS: 100**

Q. No.	**SECTION A ($5 \times 2 = 10$)** **Answer ANY FIVE questions**	CO	KL
1.	Evaluate $\int_0^{\pi} \frac{dx}{5+4\cos x}$.	1	1
2.	Find the integral $\int \frac{dx}{x+\sqrt{x^2-1}}$.	1	1
3.	Define improper integrals.	1	1
4.	Define Jacobian of two variables and three variables.	1	1
5.	Evaluate $\int_0^{\infty} e^{-x^2}\, dx$.	1	1
6.	Define Gamma and Beta function.	1	1

Q. No.	**SECTION B ($10 \times 1 = 10$)** **Answer ALL questions**	CO	KL
7.	Which of the following is an example of an irrational function a. $f(x) = 2x + \sqrt{3}$ b. $f(x) = \frac{1}{x^2+1}$ c. $f(x) = \frac{\sqrt{x}}{2}$ d. $f(x) = \sin x$	2	2
8.	What is the integral of $\int (x + \sqrt{5})dx$ a. $\frac{x^2}{2} + \sqrt{5}x + C$ b. $\frac{x^2}{2} + \frac{2}{3}\sqrt{5}x + C$ c. $\frac{x^2}{2} + \frac{2}{3}\sqrt{5}x^2 + C$ d. $\frac{x^2}{2} - \frac{2}{3}\sqrt{5}x^2 + C$	2	2
9.	If $f(x) \geq g(x) \geq 0$ on the interval $[a, \infty]$ then if $\int_a^{\infty} f(x)dx$ converges then $\int_a^{\infty} g(x)dx$ is a. Convergent b. Divergent c. Oscillating d. Cannot determine	2	2

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10.	If $a > 0$ then $\int_a^\infty \frac{1}{x^p} dx$ is convergent if a. $p > 1$ b. $p < 1$ c. $p = 1$ d. $p \leq 1$	2	2
11.	The integral $\int_1^\infty \frac{1}{x} dx$ is a. Convergent b. Divergent c. Oscillating d. Cannot determine	2	2
12.	Which of the following represents a polar coordinate transformation in the double integral a. $x = r\cos\theta \ ; y = r\sin\theta$ b. $r = x\cos\theta + y\sin\theta$ c. $x = r\sin\theta \ ; y = r\cos\theta$ d. $r = x^2 + y^2$	2	2
13.	$\int_0^{\frac{\pi}{2}} sin^7\theta \, cos^5\theta d\theta$ is a. $\frac{1}{120}$ b. $\frac{1}{60}$ c. 120 d. 60	2	2
14.	The value of $\Gamma\left(\frac{1}{2}\right)$ a. π b. $\frac{1}{2}$ c. $\sqrt{\pi}$ d. $\sqrt{\frac{1}{2}}$	2	2
15.	$\Gamma(n+1)$ a. $n\Gamma(n)$ b. $n!\, n\Gamma(n)$ c. $n!\,\Gamma(n)$ d. n	2	2
16.	$\Gamma(n)$ converges for a. $n > 0$ b. $n < 0$ c. $n = 0$ d. $n \neq 0$	2	2

Q. No.	**SECTION C ($2 \times 15 = 30$)** **Answer ANY TWO questions**	CO	KL
17.	Evaluate $\int \frac{dx}{(1+x^2)\sqrt{1-x^2}}$	3	3
18.	Evaluate $\iint (x^2 + y^2)dxdy$ over the region for which $x, y \geq 0$ and $x + y \leq 1$.	3	3
19.	By changing into polar coordinates Evaluate the integral $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} (x^2 + y^2)dxdy$	3	3
20.	Determine if the following integral is convergent or divergent. If it is convergent find its value $\int_0^3 \frac{1}{\sqrt{3-x}} dx$	3	3

Q. No.	**SECTION D ($2 \times 15 = 30$)** **Answer ANY TWO questions**	CO	KL
21.	Given that $x + y = u, y = uv$, change the variables to u, v in the integral $\iint [xy(1 - x - y)]^{\frac{1}{2}} dxdy$ taken over the area of the triangle with sides $x = 0, y = 0, x + y = 1$ and evaluate it	4	4
22.	Find the area of the cardiod using double integral	4	4
23.	Find the volume bounded by the cylinder $x^2 + y^2 = 4$, the planes $y + z = 4$ and $z = 0$.	4	4
24.	Express $\int_0^1 x^m (1 - x^n)^p \, dx$ in terms of Gamma functions and evaluate the integral $\int_0^1 x^5 (1 - x^3)^{10} dx$	4	4

Q. No.	**SECTION E ($2 \times 10 = 20$)** **Answer ANY TWO questions**	CO	KL
25.	Evaluate a. $\int \sqrt{(x-3)(7-x)} \, dx$ b. $\int \frac{dx}{\sqrt{(x-\alpha)(\beta-x)}} \, (\beta > \alpha)$	5	5
26.	Evaluate $\iiint_E \, zdV$, where E is the solid tetrahedron bounded by the four planes $x = 0, y = 0, z = 0$ and $x + y + z = 1$.	5	5
27.	By transforming into polar co-ordinates evaluate $\iint \frac{x^2y^2}{x^2+y^2} dxdy$ over the annular region between the circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = b^2 \quad (b > a)$	5	5
28.	State and prove relation between beta and gamma functions.	5	5