STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2019-20 & thereafter)

B. Sc. DEGREE EXAMINATION, APRIL 2024
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE	**:**	**MAJOR CORE**
PAPER	**:**	**VECTOR SPACES AND LINEAR TRANSFORMATIONS**
SUBJECT CODE	**:**	**19MT/MC/VL64**
TIME	**:**	**3 HOURS** — **MAX. MARKS : 100**

SECTION – A

ANSWER ANY TEN QUESTIONS. **(10 × 2 = 20)**

1. Define a subspace of a vector space.
2. Determine whether the following set of vectors {(-1,2),(2,-4)} are linearly dependent (or) not.
3. State Orthogonal matrix theorem.
4. Define Kernel and range of a linear transformation.
5. Define matrix transformation.
6. Define affine transformation.
7. The following matrices A and C ,C is invertible. Use similarity transformation $C^{-1}AC$ to transform A into a matrix B.
   $A = \begin{bmatrix} 7 & -10 \\ 3 & -4 \end{bmatrix}$ $C = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$
8. When a matrix is said to be orthogonally diagonalizable?
9. Define an inner product space.
10. Define an angle between two vectors.
11. Define orthogonal vectors.
12. State Rank-nullity theorem.

SECTION –B

ANSWER ANY FIVE QUESTIONS. **(5 × 8 = 40)**

13. Show that the vectors (1,2,0),(0,1,-1),(1,1,2) span R^3 .
14. Show that the row space and column space of a matrix A have the same dimensions.
15. Prove that a linear transformation T is one to one if and only if the kernel is the zero vector.
16. Show that the following matrix.
    a) $A=\begin{bmatrix} -4 & -6 \\ 3 & 5 \end{bmatrix}$ is diagonalizable .
    b) Find a diagonal matrix D that is similar to A.
17. Find the least square parabola for the following data points (1,7),(2,2),(3,1),(4,3).
18. Let $v_1, \dots, v_m$ be vectors in a vector space V. Let U be the set consisting of all linear combination of $v_1, \dots, v_m$. Then prove that U is a subspace of V spanned by the vectors $v_1, \dots, v_m$. (U is said to be the vector space generated by $v_1, \dots, v_m$. It is denoted span$\{v_1, \dots, v_m\}$.)
19. Show that the following matrix $A=\begin{bmatrix} 5 & -3 \\ 3 & -1 \end{bmatrix}$ is not diagonalizable.

SECTION –C

ANSWER ANY TWO QUESTIONS. **($2 \times 20 = 40$)**

20. a) Let the vectors $v_1, \dots, v_n$ span a vector space V. Prove that each vector in V can be expressed uniquely as a linear combination of these vectors if and only if the vectors are linearly independent.

b) Let $\{u_1, \dots, u_n\}$ be an orthonormal basis for a vector space V. Let v be a vector in V, then prove that v can be written as a linear combination of these basis vectors as follows:

$$v = (v.u_1)u_1 + (v.u_2)u_2 + \dots. + (v.u_n)u_n$$

21. a) State and prove the Gram-Schmidt Orthogonalization Process.

b) Let T be a linear transformation of $R^n \rightarrow R^n$. Prove that T is invertible if and only if it is nonsingular and also prove the inverse is unique and linear.

22. (a) Let A be an n x n matrix. Then prove the following

i) If A has n linearly independent eigen vectors then it is diagonalizable and another matrix C whose columns consist of n linearly independent eigen vectors can be used in a similarity transformation $C^{-1}AC$ to give a diagonal matrix D. The diagonal elements of D will be the eigen values of A.

ii) If A is diagonalize then it has n linearly independent eigen vectors.

(b) Find the fourth Fourier approximation to $f(x) = x$ over the interval $[-\pi, \pi]$.