STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted from the academic year 2019-20 & thereafter)

B. Sc. DEGREE EXAMINATION, APRIL 2024
BRANCH I – MATHEMATICS
FOURTH SEMESTER

COURSE	**:**	**MAJOR CORE**
PAPER	**:**	**SEQUENCE AND SERIES**
SUBJECT CODE	**:**	**19MT/MC/SS44**
TIME	**:**	**3 HOURS**

MAX. MARKS : 100

SECTION – A

ANSWER ANY TEN QUESTIONS: (10×2=20)

1. Define composition of functions.
2. Define characteristic function.
3. State least upper bound axiom.
4. Define a sequence of real numbers.
5. Define limit of a sequence.
6. Define monotone sequence.
7. Define limit superior of a sequence.
8. Give an example of conditionally convergent series.
9. State the ratio test for absolute convergence.
10. State Cauchy condensation test.
11. Define Fourier series.
12. Define an even function.

SECTION – B

ANSWER ANY FIVE QUESTIONS: (5×8=40)

13. Prove that the set of all rational numbers is countable.
14. Prove that if $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are sequences of real numbers, if $\lim_{n\to\infty} s_n$=L, and if $\lim_{n\to\infty} t_n$= M, then $\lim_{n\to\infty}(s_n + t_n)$= L+M.
15. Prove that if the sequence of real numbers $\{s_n\}_{n=1}^{\infty}$ is convergent, then $\{s_n\}_{n=1}^{\infty}$ is bounded.
16. Prove that if $\sum a_n$ is a convergent series then $\lim_{n\to\infty} a_n$= 0.
17. Show that the series $\sum_{n=1}^{\infty} 1/n$ is divergent.
18. State and prove comparison test.
19. Find a sine series for $f(x) = c$ in the range 0 to π.

SECTION – C

ANSWER ANY TWO QUESTIONS: **(2×20=40)**

20. a) Show that the set $[0,1] = \{x \mid 0 \le x \le 1\}$ is uncountable.

    b) Prove that the sequence $\{\left(1+\frac{1}{n}\right)^n\}_{n=1}^{\infty}$ converges.

21. a) If $\{s_n\}_{n=1}^{\infty}$ is a Cauchy sequence of real numbers, then prove that $\{s_n\}_{n=1}^{\infty}$ is convergent.

    b) If $\{a_n\}_{n=1}^{\infty}$ is a sequence of positive numbers such that

    a) $a_1 \ge a_2 \ge \ldots \ge a_n \ge a_{n+1} \ge \ldots$ [i.e. $\{a_n\}_{n=1}^{\infty}$ is non- increasing]

    b) $\lim_{n\to\infty} a_n = 0$

    then prove that the alternating series $\sum_{n=1}^{\infty}(-1)^{n+1} a_n$ is convergent.

22. a) State and prove Abel's Lemma.

    b) Determine the Fourier series expansion of the function $f(x) = x$, in the interval $-\pi < x < \pi$.