STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted from the academic year 2019–20 & thereafter)

B. Sc. DEGREE EXAMINATION, APRIL 2024
BRANCH I – MATHEMATICS
SIXTH SEMESTER

COURSE : MAJOR CORE
PAPER : PRINCIPLES OF COMPLEX ANALYSIS
SUBJECT CODE : 19MT/MC/CA65
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION-A

ANSWER ANY TEN QUESTIONS: **10 × 2 = 20**

1. Define analytic function.
2. Under the mapping $w = e^z$, discuss the transformation of the line $y = 0$.
3. Find the value of z such that $e^z = 1 + \sqrt{3}\, i$.
4. Evaluate using Cauchy integral formula $\frac{1}{2\pi i}\int_C \frac{z^2+5}{z-3} dz$ where $C\ is\ |z| = 4$.
5. Determine the angle of rotation and scale factor at the point $z = 1 + i$, under the mapping $w = z^2$.
6. Find the zeros of the following function $f(z) = \frac{z^3-1}{z^3+1}$.
7. State CR equations.
8. Define Harmonic function.
9. State Liouville's theorem.
10. State Maclaurin's series.
11. Define simple pole.
12. Define Removable singularity.

SECTION-B

ANSWER ANY FIVE QUESTIONS: **5 × 8 = 40**

13. Derive CR equations in polar coordinates.
14. State and prove the Cauchy Integral formula.

15. Find the linear fractional transformation that maps the points $z_1 = 2, z_2 = i, z_3 = -2$ onto the points $w_1 = 1, w_2 = i, w_3 = -1$.

16. Find the series expansion of the function $f(z) = \frac{1+2z^2}{z^3+z^5}$.

17 State and prove Cauchy's residue theorem.

18 State and prove Rouche's theorem.

19 Find the residue of $\frac{e^z}{z^2(z^2+9)}$ at its poles.

SECTION-C

ANSWER ANY TWO QUESTIONS: **2 × 20 = 40**

20. (a) Derive the sufficient condition for differentiability of a complex function $f(z)$.

    (b) Find the poles of the function $f(z) = \cot z$.

21. Derive the mappings of the upper half plane.

22. (a) State and prove the maximum modulus theorem.

    (b) Explain an application of conformal mapping briefly.