STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 600 086.
(For candidates admitted during the academic year 2023 – 2024

M.Sc., DEGREE EXAMINATION NOVEMBER 2023
PHYSICS
FIRST SEMESTER

COURSE : MAJOR CORE
PAPER : STATISTICAL MECHANICS
SUBJECT CODE : 23PH/PC/SM14
TIME : 3 HOURS **MAX. MARKS : 100**

Q. No.	SECTION A Answer ALL the Questions (10 x 3 = 30 marks)	CO	KL
1	Define the term equal-e-priori probability. Calculate the equal-e-priori probability of a system with 10^{12} microstates.	CO1	K1
2	Find the volume of a cell in the phase space of a system of 10 non-interacting linear harmonic oscillators.	CO1	K1
3	What is the thermodynamic probability associated with an element of the canonical ensemble with average energy E_i.	CO1	K1
4	Determine the canonical partition function of a three level system with energy values given by $\varepsilon, 0 - \varepsilon$. From that determine the probability associated with the level with energy 0.	CO2	K2
5	What is a Slater determinant? Write down the Slater determinant of a two particle Fermion system.	CO2	K2
6	Under what conditions a system of identical particles can be treated classically?	CO2	K2
7	Why do Bosons condense while Fermions do not?	CO2	K2
8	Define chemical potential. Why should it be negative for Bosons and zero for photons?	CO3	K3
9	Define Fermi temperature. Calculate the Fermi temperature of a system with a Fermi energy of 3.2 eV.	CO3	K3
10	Why does electronic heat capacity dominate over lattice heat capacity at very small temperatures?	CO3	K3
Q. No.	**SECTION B (30 marks)**	**CO**	**KL**
	PART A **(PROBLEM SECTION)** **Answer any TWO Questions: (2 x 5 = 10 marks)**		
11	Construct the density matrices of two systems – one consisting of a superposition of vacuum state $\|0\rangle$ and $\|1\rangle$ and the other a homogeneous mix of $\|0\rangle$ and $\|1\rangle$ of equal weight. Establish that $\rho^2 = \rho$ for pure states and $\rho^2 < 1$ for mixed states.	CO3	K3
12	Consider a Boson system with four energy levels of energies, $0, \varepsilon, 2\varepsilon, 3\varepsilon$. Their degeneracies are respectively $g_0 = 1, g_\varepsilon = 2, g_{2\varepsilon} = 3, g_{3\varepsilon} = 4$. If there are 10 particles in the system, find the number of ways these particles can be distributed among the energy levels to have a total energy of 12ε. Find the most probable distribution $\{n_i\}$. Also determine the entropy corresponding to that distribution.	CO3	K3
13	If $E_1 = 1.8\, eV$ is the ground state energy of a system of 10 electrons in a one dimensional box of width a, calculate its Fermi energy, Fermi momentum and average energy of the system.	CO3	K3

	PART B **Answer any FOUR Questions: (4 x 5 = 20 marks)**		
14	Establish that entropy is an extensive property of a thermodynamic system. Also prove that the expression $S = -k\sum_i P_i \ln P_i$ reduces to $S = k \ln \Omega$ for microcanonical ensemble.	**CO4**	**K4**
15	The Hamiltonian of a system of N non-interacting classical particles is $\widehat{H} = \sum_{i=1}^{3N}(aP_i^2 + bq_i^2)$ where $P_i's$ and $q_i's$ are canonically conjugate momenta and position coordinates. Evaluate the total average energy of the system at absolute temperature T using equipartition theorem. From this, obtain Dulong-Petit's law of specific heat capacity.	**CO4**	**K4**
16	Determine the fluctuation in the number of particle in a grand canonical ensemble and show that the grand canonical ensemble of a large system approximates the canonical ensemble.	**CO4**	**K4**
17	Derive the BE distribution function and plot it for three different temperatures. Describe its behavior when the temperature is very close to absolute zero.	**CO4**	**K4**
18	Determine the thermodynamical properties of a system of N Fermions at absolute zero.	**CO4**	**K4**
Q. No.	**SECTION C** **Answer the following: (2 x 20 = 40 marks)**	**CO**	**KL**
19	a) Consider an isolated system of N particles of a classical ideal gas. Derive its equation of states by the microcanonical ensemble formalism.	**CO5**	**K5**
	b) Describe Gibb's paradox and correct Boltzmann counting. Recount the number of microstates factoring in Boltzmann counting and estimate the corrected expression for the entropy of the system.	**CO5**	**K5**
	(OR)		
	c) Consider a system of N non-interacting particles with rotational and vibrational degrees of freedom in addition to translational degree of freedom. Evaluate the partition function of the system.	**CO5**	**K5**
	d) Express the temperature and entropy of N non-interacting magnetic dipoles in an external magnetic field of strength H, in terms of its internal energy and bring out the concept of negative temperature.	**CO5**	**K5**
20	a) List out the assumptions of Einstein's theory of specific heat capacity and estimate the specific heat capacity of a monoatomic crystalline solid. Point out its limitations.	**CO5**	**K5**
	b) Modify Einstein's theory of specific heat capacity to obtain Debye's T^3 law.	**CO5**	**K5**
	(OR)		
	c) Treating the free electrons in a white dwarf as a fully degenerate non-relativistic Fermi gas establish that the size of the star decreases as its mass increases.	**CO5**	**K5**
	d) Estimate the maximum mass of a star for it to become a white dwarf.	**CO5**	**K5**

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