STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI – 86
(For candidates admitted from the academic year 2023 – 2024 and thereafter)

M.Sc. DEGREE EXAMINATION, NOVEMBER 2023
BRANCH III - PHYSICS
THIRD SEMESTER

COURSE : MAJOR CORE
PAPER : QUANTUM MECHANICS I
SUBJECT CODE: 19PH /PC/QM34
TIME : 3 HOURS **MAX. MARKS: 100**

SECTION A

Answer ALL the Questions **(10 x 3 = 30 marks)**

1. Define a linear vector space. What are the necessary conditions for a set of vectors to form the basis vectors?
2. An operator $\hat{A}$ is defined as $\hat{A}\psi = \frac{d\psi}{dx} + a\psi$. Check if it a linear operator.
3. Find the eigenvalues of the operator $\hat{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$
4. Calculate the Larmor frequency and radius of the orbit of an electron entering a magnetic field of intensity 3.2 gauss perpendicular to its velocity vector of magnitude $1.6 \times 10^7\ m - s^{-1}$.
5. Indicate why the effect of a weak electric field on atomic energy levels (Stark effect) can be found only for the ground state of Hydrogen atom when using non-degenerate perturbation theory.
6. Using the variation function $\psi = Nx(L^2 - x^2)$ find the energy of a particle in a box with rigid walls.
7. For a system with two angular momenta, $j_1 = j_2 = \frac{1}{2}$, what is the dimension of the Hilbert space? List the two sets of basis vectors that can span this space.
8. Find the form of the Pauli matrix $\hat{\sigma}_z$ in the basis $|1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$, $|2\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix}$
9. Define the term differential scattering cross section. Relate it to the total scattering cross section.
10. Identify the effect of an extended scatterer in the outcome of a scattering experiment.

SECTION B

Answer any FIVE Questions **(5 x 5 = 25 marks)**

11. The wave function of a particle at time $t = 0$ is given by $\psi(q, 0)$ is given by $(2\alpha)^{1/4} e^{(-\pi\alpha q^2 + ik_0 q)}$ where k_0 is a constant. Determine $\psi(q, t)$.
12. What are Fock states? Determine how the annihilation operator $\hat{a}$ affects these states and from that obtain the matrix form of the operator.
13. The potential energy of a linear oscillator is given by $V(x) = \frac{1}{2}\, m\omega^2 x^2 + bx^4$. Applying perturbation theory for non-degenerate levels, find the first order energy correction to the n^{th} level.

14. Evaluate the commutators $[\hat{L}_z, \hat{L}_+]$ Using this, evaluate $\hat{L}_+|l,m\rangle$ where $|l,m\rangle$ are the eigen-kets of $\hat{L}_z$.
15. With necessary mathematical formalism, describe how the spin-orbit interaction gives rise to the doublet energy levels.
16. Consider two quantum states $|\psi_1\rangle = 2i|\varphi_1\rangle + |\varphi_2\rangle - a|\varphi_3\rangle + 4|\varphi_4\rangle$ and $|\psi_2\rangle = 3|\varphi_1\rangle - i|\varphi_2\rangle + 5|\varphi_3\rangle - |\varphi_4\rangle$ where $|\varphi_i\rangle$; $i \rightarrow 1,2,3,4$ are orthogonal states and a is a scalar. Find the value of a such that $|\psi_1\rangle$ and $|\psi_2\rangle$ are orthogonal to each other.
17. If $\vec{A}$ and $\vec{B}$ are arbitrary vectors, show that, $(\vec{\sigma}.\vec{A})(\vec{\sigma}.\vec{B}) = \vec{A}.\vec{B} + i\vec{\sigma}.(\vec{A} \times \vec{B})$ where $\vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$.

SECTION C

Answer any THREE Questions **(3 x 15 = 45 marks)**

18. Setup and solve the Schrodinger wave equation for Hydrogen atom and obtain the energy eigenvalues and normalized eigen-functions.
19. i)The Hamiltonian of a two level system is given by $\hat{H} = a\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$.

    ii)Find the energy eigenvalues and the normalized eigenkets of the Hamiltonian. (7) Obtain the unitary matrix that diagonalises this Hamiltonian and demonstrate the diagonalization process. (8)
20. Formulate perturbation theory for degenerate levels and apply it to explain Stark effect in the $n = 2$ level of Hydrogen atom. Point out the effect of perturbation on the degeneracy of the state.

    (7+8)
21. i)Represent the angular momentum operators $\hat{J}_z, \hat{J}_+, \hat{J}_-, \hat{J}_x, \hat{J}_y$ as matrices in the standard basis (eigen-kets of $\hat{J}_z$) for $l = 1$. Show how $\hat{J}_x$ and $\hat{J}_y$ transform these basis vectors. (10)

    ii. List out the selection rules used in the process of addition of two angular momenta $\vec{J}_1$ and $\vec{J}_2$ and calculate the Clebsch-Gordan coefficients for the addition of $j_1 = \frac{1}{2}, \ j_2 = \frac{1}{2}$ using the operator method. (5)
22. Applying the partial wave analysis, deduce optical theorem for the elastic scattering of plane waves by a central potential.

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