STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2023 – 24)

M. Sc. DEGREE EXAMINATION, NOVEMBER 2023
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : CORE
PAPER : REAL ANALYSIS
SUBJECT CODE : 23MT/PC/RA14
TIME : 3 HOURS **MAX. MARKS : 100**

Q. No.	SECTION A ($5 \times 2 = 10$) Answer ALL questions	CO	KL
1.	State Lindelof covering theorem.	1	1
2.	Define total variation of a function $\boldsymbol{g}$ on the interval $[c, d]$.	1	1
3.	Describe upper and lower Stieltjes sums of the function h with respect to β for a partition Q.	1	1
4.	Write the Taylor's formula for functions from R^n to R^1.	1	1
5.	State the inverse function theorem.	1	1

Q. No.	SECTION B ($10 \times 1 = 10$) Answer ALL questions	CO	KL
6.	The set of rational number has _______________ as an accumulation point. (a) 0 and 1 only (b) every complex number (c) every real number (d) 0 only	2	2
7.	The collection of all intervals of the form $1/n < x < 2/n, \ (n = 2, 3, 4...)$ is ______________of the interval $0 < x < 1$ (a) an uncountable open covering (b) a countable open covering (c) a countable closed covering (d) an uncountable closed covering	2	2
8.	The total variation of the function $\sin x$ on the interval $[0, 2\pi]$ is (a) 1 (b) 2 (c) 2π (d) 4	2	2
9.	What is norm of partition {0, 3, 3.1, 3.2,8, 10} of interval [0, 10]. (a) 10 (b) 3 (c) 4.8 (d) 4.5	2	2
10.	Which theorem guarantees the existence of Riemann-Stieltjes integrals for functions that are continuous on a closed interval? (a) Fundamental Theorem of integral Calculus (b) Riemann-Sums Theorem (c) Lebesgue's Dominated Convergence Theorem (d) The Mean Value Theorem	2	2

11.	If a partition P* finer than P, then (a) $U(P^*, f, \alpha) \geq U(P, f, \alpha)$ (b) $U(P^*, f, \alpha) \leq U(P, f, \alpha)$ (c) $L(P^*, f, \alpha) \leq L(P, f, \alpha)$ (d) Non of the above are true.	2	2
12.	The total derivative of a linear function is ______ (a) the function itself (b) a variable (c) a non-linear function (d) undefined	2	2
13.	What does the directional derivative of a multivariable function study? (a) The rate of change of the function in the x-direction. (b) The rate of change of the function in the y-direction. (c) The rate of change of the function in an arbitrary direction. (d) The rate of change of the function at a specific point.	2	2
14.	Let A be an open subset of R^n and assume that $f : A \to R^n$ has continuous partial derivative $D_j f_i$ on A. If ___________, then f is an open mapping. (a) $J_f(x) = 0$ for some x in A (b) $J_f(x) = 0$ for all x in A (c) $J_f(x) \neq 0$ for some x in A (d) $J_f(x) \neq 0$ for all x in A	2	2
15.	What is the meaning of a function has a local extremum? (a) The function has local maximum (b) The function has either local maximum or local minimum (c) The function has neither local maximum nor local minimum (d) The function has local minimum.	2	2

Q. No.	SECTION C ($2 \times 15 = 30$) Answer ANY TWO questions	CO	KL
16.	(a) Prove that the intersection of finite collection of open sets is open. (8) (b) Prove that every uncountable closed set F in R^n can be expressed in the form $F = A \cup B$, where A is perfect and B is countable. (7)	3	3
17.	(a) If f is continuous on $[a, b]$ and if f' exists and is bounded in the interior, say $\lvert f'(x) \rvert \leq A$ for all x in (a, b), then prove that f is of bounded variation on $[a, b]$. (7) (b) If $f \in R(\alpha)$ and $f \in R(\beta)$ on $[a, b]$, then prove that $f \in R(c\alpha + d\beta)$ on $[a, b]$, where c and d are any two constants. (8)	3	3

18.	Discuss the interchanging order of integration in Riemann-Stieltjes integration. (15)	3	3
19.	(a) State and prove the sufficient condition for differentiability. (10) (b) Let f be a function from R^2 to R^3 defined by equation $f(x,y)=(\sin x\cos y, \sin x\sin y, \cos x\cos y)$. Determine the Jacobian matrix. (5)	3	3

Q. No.	SECTION D $(2 \times 15 = 30)$ Answer ANY TWO questions	CO	KL
20.	Let S be a subset of R^n. then prove that the following statements are equivalent. (a) S is compact (b) S is closed and bounded (c) Every infinite subset of S has an accumulation point in S (15)	4	4
21.	(a) Check whether the function $f(x)=x^2\sin(1/x)$ *if* $x\neq 0$, $f(0)=0$ is of bounded variation or not on [0,1]. (8) (b) Write the definitions of Riemann – Steieltjes integral and prove that using the defintion $\int_c^d d\gamma(x)=\gamma(d)-\gamma(c)$. (7)	4	4
22.	(a) Show that the function, defined on an interval [0,1] $f(x)=1$, if x is rational, $f(x)=0$, if x is irrational is not Riemann integrable on [0,1]. (5) (b) State and prove the necessary conditions for the existence of Riemann-Stieltjes integrals. (10)	4	4
23.	(a) Assume that $f=(f_1,\ldots,f_n)$ has continuous partial derivatives $D_j f_i$ on an open set S in R^n, and that the Jacobian determinant $J_f(a)\neq 0$ for some point 'a' in S then prove that there is an n −ball $B(a)$ on which f is one-to-one. (7) (b) Find and classify the extreme values (if any) of the functions defined by the following equations : (i) $f(x,y)=y^2+x^2y+x^4$ (ii) $f(x,y)=y^2-x^3$. (4+4)	4	4

Q. No.	**SECTION E $(2 \times 10 = 20)$** **Answer ANY TWO questions**	**CO**	**KL**
24.	State and prove Bolzano-Weierstrass theorem.	5	5
25.	(a) Let f be of bounded variation on $[a, b]$, and assume that $c \in (a,b)$, then prove that f is of bounded variation on $[a, c]$ and on $[c, b]$, we have $V_f(a,b) = V_f(a,c) + V_f(c,b)$. (5) (b) Discuss the change of variable in a Riemann Stieltjes integral. (5)	5	5
26.	(a) Prove that every closed set in R^1 is the intersection of a countable collection of open sets. (5) (b) Prove that there is no real valued function f such that $f'(c;u) > 0$ for a fixed c in $\boldsymbol{R^n}$ and every non-zero vector $\mathbf{u}$ in R^n. Give an example such that $f'(c;u) > 0$ for a fixed direction $\mathbf{u}$ and every c in $\boldsymbol{R^n}$. (5)	5	5
27.	State and prove the second derivative test for extrema.	5	5

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