STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2023 – 24)

M. Sc. DEGREE EXAMINATION, NOVEMBER 2023
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE	:	**CORE**
PAPER	:	**ORDINARY DIFFERENTIAL EQUATIONS**
SUBJECT CODE	:	**23MT/PC/OD14**
TIME	:	**3 HOURS** **MAX. MARKS : 100**

Q. No.	**SECTION A** $(\mathbf{5 \times 2 = 10})$ **Answer ALL questions**	**CO**	**KL**
1.	Define linear dependence.	1	1
2.	What is meant by fundamental matrix?	1	1
3.	State any two properties of Bessel's function.	1	1
4.	Write down the Lipschitz condition.	1	1
5.	Define regular linear boundary value problem.	1	1

Q. No.	**SECTION B** $(\mathbf{10 \times 1 = 10})$ **Answer ALL questions**	**CO**	**KL**
6.	The Wronskian of 1, t and t^2 is (a) 1 (b) -1 (c) 2 (d) -2	2	2
7.	The second approximate solution of $x' = x^2, x(0) = 1$, as per Picard's successive approximation method is (a) 1 (b) $1 + t$ (c) $1 - t$ (d) t^2	2	2
8.	When a linear equation $x''' - 4x'' + 10x' - 2x = 0$ is transformed to linear system $x' = Ax$, where A is (a) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -10 & 2 \end{bmatrix}$ (b) $\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & -10 & 4 \end{bmatrix}$ (c) $\begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 2 \\ 2 & -5 & 1 \end{bmatrix}$ (d) $\begin{bmatrix} 0 & 2 & 0 \\ 0 & 0 & 2 \\ 1 & -5 & 2 \end{bmatrix}$	2	2
9.	Which of the following is not a regular singular point of the equation $t(t-1)^2(t+3)x'' + t^2x' - (t^2 + t - 1)x = 0$? (a)1 (b) 0 (c) -2 (d) none of these	2	2
10.	When p is an integer, $J_{-P}(t) =$ (a) $J_P(t)$ (b) $pJ_P(t)$ (c) $(-1)^p J_P(t)$ (d) $-J_P(t)$	2	2
11.	Find the general solution of $x'' - 2x' - 3x = 0$. (a) $C_1e^{3t} + C_2e^{-t}$ (b) $C_1e^{-3t} + C_2e^{t}$ (c) $C_1e^{-3t} + C_2e^{-t}$ (d) none of these	2	2
12.	Let f be a periodic with period ω. A solution x of $x' = Ax + f(t)$, $t \in (-\infty, \infty)$ is periodic of the period ω if and only if (a) $x(0) = x(1)$ (b) $x(0) = x(\omega)$ (c) $x(1) = x(\omega)$ (d) $x(-\infty) = x(\infty)$	2	2
13.	Let $f: [t_0, \infty] \to [0, \infty]$ be a continuous function and $k > 0$ be a constant. If $f(t) \leq k \int_{t_0}^{t} f(s)ds$, $t \geq t_0$, then which of the following holds? (a) $f(t) > 0$ (b) $f(t) < 0$ (c) $f(t) = 0$ (d) none	2	2

14.	All the eigenvalues of Strum-Liouville problem are (a) real (b) complex (c) mixed real and complex (d) none	2	2
15.	The boundary conditions $x(A) = x(B)$ and $x'(A) = x'(B)$ are known as (a) initial (b) periodic (c) singular (d) non singular	2	2

Q. No.	**SECTION C** $(\mathbf{2 \times 15 = 30})$ **Answer ANY TWO questions**	**CO**	**KL**
16.	Let $b_1, b_2, \dots, b_n : I \to \mathbb{R}$ be real continuous functions in the n-th order homogeneous differential equation $L(x) = 0$. Prove that $\varphi_1, \varphi_2, \dots, \varphi_n$ are n linearly independent solutions of $L(x) = 0$ on I iff the Wronskian of $\varphi_1, \varphi_2, \dots, \varphi_n$ is non-zero for every $t \in I$. In addition, apply this to the equation $x'' - \frac{2}{t^2}x = 0,\ 0 < t < \infty$.	3	3
17.	Formulate a unique solution for a linear system $x' = A(t)x,\ x(t_0) = x_0$.	3	3
18.	Obtain the linearly independent solutions of Legendre equation.	3	3
19.	Derive the Picard's theorem.	3	3

Q. No.	**SECTION D** $(\mathbf{2 \times 15 = 30})$ **Answer ANY TWO questions**	**CO**	**KL**
20.	Consider a linear system $x' = A(t)x$ where $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, $A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 6 & -11 & 6 \end{bmatrix}$. Determine the fundamental matrix.	4	4
21.	Explain the existence of solution of initial first order differential equation in the large.	4	4
22.	Derive the generating function and integral representation of Bessel function.	4	4
23.	Prove that $x(t)$ is a solution of $L(x(t)) + f(t) = 0$ if and only if $x(t) = \int_a^b G(t,s)f(s)ds$ where $G(t,s)$ Green's function.	4	4

Q. No.	**SECTION E** $(\mathbf{2 \times 10 = 20})$ **Answer ANY TWO questions**	**CO**	**KL**
24.	Explain the Abel's formula.	5	5
25.	Let $x' = A(t)x$ be a linear system where $A: I \to M_n(R)$ is continuous. Suppose a matrix Φ satisfies the system, establish $(\det \Phi)' = (tr\, A)(\det \Phi)$.	5	5
26.	Derive the orthogonal property of Legendre polynomial.	5	5
27.	Evaluate the solution of the equation $x' = -x,\ x(0) = 1,\ t \geq 0$, by Picard's successive approximation method and verify with analytical method.	5	5