Solving Differential Equations

⟨Your Name⟩

1 Introduction

Differential equations are a fundamental topic in mathematics and science. In this document, we will solve a simple first & second order ordinary differential equation (ODE).

2 First-Order ODE

Consider the following first-order ODE:

$$\frac{dy}{dx} = 2x \tag{1}$$

We can solve this ODE using separation of variables[1]. Separating the variables y and x, we get:

$$\frac{dy}{y} = 2x \, dx$$

Now, we integrate both sides:

$$\int \frac{1}{y} \, dy = \int 2x \, dx$$
$$\ln|y| = x^2 + C \tag{2}$$

Where C is the constant of integration. We can solve for y:

$$|y| = e^{x^2 + C} \tag{3}$$

3 Second-Order ODE

Let's consider a second-order ODE with initial conditions. For example:

$$y''(t) + 4y(t) = 0, \quad y(0) = 1, \quad y'(0) = 0 \tag{4}$$

This is a simple linear homogeneous ODE. The general solution[2] of equation (4) can be written as:

$$y(t) = c_1 \cos(2t) + c_2 \sin(2t)$$

References

[1] William E. Boyce and Richard C. DiPrima. (2017). *Elementary Differential Equations and Boundary Value Problems*, Wiley Publ.

[2] Stanley J. Farlow. (1993). *Partial Differential Equations for Scientists and Engineers*, Dover Publ.