STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019 – 20 and thereafter)

M. Sc. DEGREE EXAMINATION, NOVEMBER 2023
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE	**:**	**CORE**
PAPER	**:**	**PARTIAL DIFFERENTIAL EQUATIONS**
SUBJECT CODE	**:**	**19MT/PC/PD34**
TIME	**:**	**3 HOURS**

MAX. MARKS : 100

SECTION – A

ANSWER ALL THE QUESTIONS: (5 × 2 = 10)

1. State Cauchy's problem for the first order partial differential equations.
2. Find the characteristics of the hyperbolic equation $3u_{xx} + 10u_{xy} + 3u_{yy} = 0$
3. Write down the Poisson equation.
4. State any two properties of Dirac delta function.
5. State the D'Alembert's solution of the one dimensional wave equation.

SECTION – B

ANSWER ANY FIVE QUESTIONS: (5 × 6 = 30)

6. Find the general solution of the differential equation $x^2 \frac{\partial z}{\partial x} + y^2 \frac{\partial z}{\partial y} = (x + y)z$
7. Reduce the Tricomi equation $u_{xx} + xu_{yy} = 0, x \neq 0$ for all x, y to canonical form.
8. Derive the Laplace equation $\nabla^2 V = 0$
9. A one-dimensional infinite region $-\infty < x < \infty$ is initially kept at zero temperature. A heat source of strength g_s units, situated at $x = \xi$ releases its heat instantaneously at time $t = \tau$. Determine the temperature in the region for $t > \tau$.
10. Derive one dimensional wave equation.
11. Obtain the solution of the wave equation $u_{tt} = c^2 u_{xx}$ under the following conditions
    (i) $u(0,t) = u(2,t) = 0$ (ii) $u(x, 0) = \sin^3 \frac{\pi x}{2}$ (iii) $u_t(x, 0) = 0$
12. Classify and transform the following equation to a canonical form
    $\sin^2(x)u_{xx} + \sin(2x)\, u_{xy} + \cos^2(x)u_{yy} = x$

SECTION – C

ANSWER ANY THREE QUESTIONS: **(3 × 20 = 60)**

13. (a) Find the solution of the equation $z = \frac{1}{2}(p^2 + q^2) + (p - x)(q - y)$ passes through the x −axis.
    (b) Find the integral surface of the linear partial differential equation $x(y^2 + z)p - y(x^2 + z)q = (x^2 - y^2)z$ which contains the straight line $x + y = 0, z = 1$.

14. (a) Find a complete integral of the partial differential equation $(p^2 + q^2)x = pz$ and deduce the solution which passes through the curve $x = 0, z^2 = 4y$.
    (b) Show that the only integral surface of the equation $2q(z - px - qy) = 1 + q^2$ which is circumscribed about the paraboloid $2x = y^2 + z^2$ is the enveloping cylinder which touches it along its section by the plane $y + 1 = 0$.

15. (a) Find the solution of the Neumann problem for a rectangle.
    (b) Determine the solution of the exterior Dirichlet problem for a circle.

16. (a) The ends A and B of a rod, 10 cm in length, are kept at temperatures 0^oC and 100^oC until the steady state condition prevails. Suddenly the temperature at the end A is increased to 20^oC, and the end B is decreased 60^oC. Find the temperature distribution in the rod at time t.
    (b) Solve the one-dimensional diffusion equation in the region $0 \le x \le \pi, t \ge 0$, subject to the conditions (i) T remains finite as $t \to \infty$ (ii) $T = 0$, if $x = 0$ and π for all t

$$(iii) \text{ At } t = 0, T = \begin{cases} x, & 0 \le x \le \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} \le x \le \pi \end{cases}$$

17. (a) Solve the initial value problem of the wave equation $u_{tt} - c^2 u_{xx} = f(x,t)$ subject to the initial conditions $u(x,0) = \eta(x), u_t(x,0) = v(x)$.
    (b) Establish the variables separable solution of vibrating string.