STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600086
(For candidates admitted during the academic year 2023-2024)
M. Sc. DEGREE EXAMINATION, NOVEMBER 2023

INFORMATION TECHNOLOGY
FIRST SEMESTER

| COURSE | $:$ | MAJOR CORE |
| :--- | ---: | :--- |
| PAPER | $:$ | DISCRETE MATHEMATICS FOR COMPUTER SCIENCE |
| SUBJECT CODE: | 23CS/PC/DM14 |  |

TIME : 3 HOURS MAX. MARKS: 100

| Q. No. | SECTION A | CO | KL |
| :---: | :---: | :---: | :---: |
|  | Answer all questions: (10 x 2=20) |  |  |
| 1. | Define Complemented Lattice. | CO1 | K1 |
| 2. | State the Principle of Mathematical Induction | CO1 | K1 |
| 3. | Let p denote "Joe eats Sweets" and q denote "Angel eats Chips" Write the proposition for <br> 1) If Joe eats Sweets, then Angel eats Chips. <br> 2) Joe eats Sweets if and only if Angel eats Chips | CO1 | K1 |
| 4. | Construct a truth table to show that $\left(\mathrm{p}^{\wedge} \mathrm{q}\right)$-> p is a tautology | CO1 | K1 |
| 5. | Enumerate the two types of Quantification. | CO1 | K1 |
| 6. | For the universal set N , Is $3 \mathrm{x}\left((\mathrm{x}-3=1)^{\wedge}(\mathrm{x}>3)\right)_{\text {true }}$ ? | CO1 | K2 |
| 7. | Interpret the Pigeonhole principle. | CO1 | K2 |
| 8. | Outline any two properties of Asymptotic Domination. | CO1 | K2 |
| 9. | Describe the Characteristics of a Tree. | CO1 | K2 |
| 10. | Find the Union and Intersection of the following graphs. | CO1 | K2 |
| Q. No. | SECTION B  <br> Answer all the questions $(4 \times 5=20)$ | CO | KL |
| 11. | a) Use mathematical induction to show the following: For any natural number $n$ such that $n>=4$, prove that $n!>n^{2}$. <br> (OR) <br> b) Apply Square Root $I$ algorithm to find the value of $\sqrt{ } 17$. | CO 2 | K3 |


| 12. | a) Use the Boolean laws to prove that (not $\left.\mathrm{p}^{\wedge} \mathrm{q}\right) \mathrm{V}\left(\mathrm{p}{ }^{\wedge}\right.$ not q) $V\left(p^{\wedge} q\right)$ is logically equivalent to the formula $p v q$. Provide a circuit equivalent to the logic. <br> (OR) <br> b) Let $x$, $y$ be elements of a Boolean algebra. Make use of the DNF for the Boolean expression ( $\mathrm{x} \wedge$ y) $\mathrm{V}\left(\operatorname{not} \mathrm{x}^{\wedge}\right.$ not y) to design a combinatorial circuit. | CO 2 | K3 |
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| 13. | a) Give an example of a universal set U and predicates P and Q such that $(\forall \mathrm{xP}(\mathrm{x})) \longrightarrow(\forall \mathrm{x} . \mathrm{Q}(\mathrm{x}))$ is true but $\forall$ $x(P(x)->Q(x))$ is false. <br> (OR) <br> b) <br> For the family tree given in the above figure identify the elements of the relations (a) IsMarriedTo, (b) IsParentOf and (c) IsSameGeneration | CO 2 | K3 |
| 14. | a) Prove the following theorem. Let $\mathrm{F}: \mathrm{X}$->Y where X and $Y$ are finite with $\|X\|=\|Y\|$. Then, $F$ is $1-1$ if and only if $F$ is onto <br> (OR) <br> b) Test the following. Let $\mathrm{m} \in \mathrm{N}$. Given m integers al, a2 .... am, there exist k and l with $0<=\mathrm{k}<1<=\mathrm{m}$ such that $\mathrm{a}_{\mathrm{k}+1}$ $+a_{k+2}++a_{1}$ is divisible by $m$. | CO3 | K4 |
| Q. No. | SECTION C  <br> Answer all the questions $(6 \times 10=60)$ | CO | KL |
| 15 | a) Modify the following into CNF <br> 1) $(P \rightarrow(Q \rightarrow R)) \rightarrow(P \rightarrow(R \rightarrow Q))$ <br> 2) $(P \rightarrow Q) \rightarrow((Q \rightarrow R) \rightarrow(P \rightarrow R))$ <br> (OR) <br> b) Theorem 1. Every formula is logically equivalent to a formula in DNF. <br> Theorem 2. Every formula is logically equivalent to a formula in CNF. <br> Apply Theorem 1 and 2 on the following formula and prove. <br> $(\operatorname{not}(\mathrm{p}->\mathrm{q}))->(\mathrm{q} \wedge$ not r$)$ | CO 2 | K3 |


| 16 | a) Prove the following using the principle of mathematical induction $\forall \mathbf{n} \in \mathrm{N}: 1.2+2.3+3.4+\ldots . .+\mathrm{n}(\mathrm{n}+1)=[\mathbf{n}(\mathbf{n}+\mathbf{1})(\mathbf{n}+\mathbf{2})$ <br> ]/3 <br> (OR) <br> b) The terms of a sequence are given recursively as $\mathrm{a}_{0}=0, \mathrm{a}_{1}$ $=2$, and $a_{n}=4\left(a_{n-1}-a_{n-2}\right)$ for $n>=2$. <br> Prove by induction that $b_{n}=n .2^{n}$ is a closed form for the sequence. That is, prove that $a_{n}=b_{n}$ for every $n € N$. | CO3 | K4 |
| :---: | :---: | :---: | :---: |
| 17 | a)) Prove that the Petersen graph shown here is nonHamiltonian. <br> (b) Prove that by removing any single vertex and its incident edges, the resulting graph is Hamiltonian. <br> (OR) <br> b)Let $G$ be a graph. Prove that $G$ is bipartite if and only if $G$ contains no odd cycle. | CO3 | K4 |
| 18 | a) Explain measuring the time complexity of an algorithm for polynomial and non-deterministic polynomial problems. <br> (OR) <br> b)How to measure the complexity of an algorithm in structured programming? Formulate with the help of counting statements. | CO4 | K5 |
| 19 | a)Portray the special types of relations with a suitable example. <br> (OR) <br> b)Write the loop invariant assertions using Bubble sort. | CO4 | K5 |
| 20 | a) Resolve the Depth First Search Algorithm to examine the vertices and the edges of the given graph. <br> (OR) <br> b) Verify that $\mathrm{O}\left(\log _{2}(\mathrm{n}!)\right)=\mathrm{O}(\mathrm{n} \log (\mathrm{n}))$. | CO5 | K6 |

