**STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086**
**(For candidates admitted during the academic year 2023–24)**

**B. Sc. DEGREE EXAMINATION, NOVEMBER 2023**
**BRANCH I - MATHEMATICS**
**FIRST SEMESTER**

**COURSE : MAJOR – CORE**
**PAPER : DIFFERENTIAL CALCULUS**
**SUBJECT CODE : 23MT/MC/DC14**
**TIME : 3 HOURS** **MAX. MARKS : 100**

| Q. No. | **SECTION A** $(5 \times 2 = 10)$<br>**Answer ANY FIVE questions** | CO | KL |
|---|---|---|---|
| 1. | If $y = \sin 3x \cos 2x$ find $y_n$. | 1 | 1 |
| 2. | Show that the radius of curvature of $y = c \cosh \frac{x}{c}$ is $\frac{y^2}{c}$. | 1 | 1 |
| 3. | Find the radius of curvature at any point $(p, r)$ of the curve $p^2 = ar$. | 1 | 1 |
| 4. | Find the envelope of family of straight lines $xcos\alpha + ysin\alpha = a$ , $\alpha$ being a parameter. | 1 | 1 |
| 5. | State the necessary conditions for extrema of functions of two variables. | 1 | 1 |
| 6. | Define a catenary. | 1 | 1 |

| Q. No. | **SECTION B** $(10 \times 1 = 10)$<br>**Answer ALL questions** | CO | KL |
|---|---|---|---|
| 7. | The $n^{th}$ derivative of a function whose numerator and denominator are both rational integral algebraic functions can be obtained by resolving the fraction into ----------<br>a) Factors b) Partial Fractions c) Product d) None of the above | 2 | 2 |
| 8. | The $n^{th}$ derivative of ---------- of two functions can be found by using Leibnitz theorem.<br>a) Sum b) Difference c) Product d) Quotient | 2 | 2 |
| 9. | An equation of a curve in terms of $s$ and $\psi$ is called ---------- equation.<br>a) Cartesian b) Parametric c) Polar d) Intrinsic | 2 | 2 |
| 10. | The locus of centre of curvature of a curve is called its ----------<br>a) Curvature b) Evolute c) Involute d) Radius of curvature | 2 | 2 |
| 11. | If each of the members of a family of curves touches a fixed curve , the $E$ is called the ---------- of the family of curves .<br>a) Evolute b) Involute c) Envelope d) None of the above | 2 | 2 |
| 12. | The evolute of a curve is envelope of its ----------<br>a) Tangents b) Normals c) Chord d) Diameter | 2 | 2 |
| 13. | The points where $f_x = 0$ , $f_y = 0$ are called ----------<br>a) Stationary points b) Saddle points<br>c) critical points d) all of the above | 2 | 2 |
| 14. | To find the extrema of function , subject to a condition we use ---------- multiplier method.<br>a) Leibnitz b) Cramer c) Hamilton d) Lagrange | 2 | 2 |
| 15. | -------- is the curve traced by a point on the circumference of a circle which rolls (without sliding) on a circle.<br>a) Cycloid b) Catenary c) Logarithmic Spiral d) Cardioid | 2 | 2 |
| 16. | Node and Cusp are classification of ---------- Point<br>a) Singular b) Isolated c) Double d) Stationary | 2 | 2 |

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| **Q. No.** | **SECTION C** $(\mathbf{2 \times 15 = 30})$<br>**Answer ANY TWO questions** | **CO** | **KL** |
|---|---|---|---|
| 17. | If $y = (sin^{-1}x)^2$ , then show that<br>(i) $(1-x^2)y_2 - xy_1 - 2 = 0$<br>(ii) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} - n^2y_n = 0$ | 3 | 3 |
| 18. | Find the envelope of family of straight lines $\frac{x}{a} + \frac{y}{b} = 1$ , where $a$ and $b$ are parameters connected by the relation $a^2 + b^2 = c^2$. | 3 | 3 |
| 19. | Is origin a double point of the curve $y^2 = 2x^2y + x^4y - 2x^4$? If so state its nature. | 3 | 3 |
| 20. | Find the evolute of $x = at^2$ , $y = 2at$. | 3 | 3 |

| **Q. No.** | **SECTION D** $(\mathbf{2 \times 15 = 30})$<br>**Answer ANY TWO questions** | **CO** | **KL** |
|---|---|---|---|
| 21. | a) Find the $n^{th}$ derivative of $\frac{x^2+1}{(x-1)(x-2)(x-3)}$.<br>b) If $x = a(\theta + \sin\theta)$ , $y = a(1 - \cos\theta)$ , find $\frac{d^2y}{dx^2}$.<br>(10+5) | 4 | 4 |
| 22. | Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. | 4 | 4 |
| 23. | Find the minimum value of $x^2 + y^2 + z^2$ subject to $2x + 3y + 5z = 30$. | 4 | 4 |
| 24. | a) Show that the chord of curvature through pole of the curve $r^m = a^m \cos m\theta$ is $\frac{2r}{m+1}$.<br>b) Find the radius of curvature at the point $(x, y)$ on the curve $y = a \log \sec\left(\frac{x}{a}\right)$. (8+7) | 4 | 4 |

| **Q. No.** | **SECTION E** $(\mathbf{2 \times 10 = 20})$<br>**Answer ANY TWO questions** | **CO** | **KL** |
|---|---|---|---|
| 25. | Find the equation of circle of curvature at the point $(3,1)$ of the curve $y = x^2 - 6x + 10$. | 5 | 5 |
| 26. | If $x = sin\theta$ , $y = sinp\theta$ , then prove that<br>(i) $(1-x^2)y_2 - xy_1 + p^2y = 0$.<br>(ii) $(1-x^2)y_{n+2} - (2n+1)xy_{n+1} + (p^2 - n^2)y_n = 0$. | 5 | 5 |
| 27. | Examine the extreme values for the function $xy(6 - x - y)$ | 5 | 5 |
| 28. | Derive the equation of cycloid and state its characteristic properties. | 5 | 5 |

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