STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted during the academic year 2023-24)

B. Sc. DEGREE EXAMINATION, NOVEMBER 2023
BRANCH I - MATHEMATICS
FIRST SEMESTER

COURSE : MAJOR – CORE
PAPER : ALGEBRA AND TRIGONOMETRY
SUBJECT CODE : 23MT/MC/AT13
TIME : 3 HOURS MAX. MARKS : 100

Q. No.	SECTION A ($5 \times 2 = 10$) Answer ANY FIVE questions	CO	KL
1.	Solve the equation $x^3 - 12x^2 + 39x - 28 = 0$ whose roots are in arithmetic progression.	1	1
2.	Change the equation $2x^4 - 3x^3 - 3x^2 - x + 2 = 0$ into another the coefficient of whose highest term will be unity.	1	1
3.	Show that the coefficient of x^n in the infinite series $1 + \frac{b+ax}{1!} + \frac{(b+ax)^2}{2!} +, \dots, + \frac{(b+ax)^n}{n!} + \cdots$ is $\frac{e^b a^n}{n!}$	1	1
4.	State Cayley Hamilton theorem.	1	1
5.	Write the expansion for $cosn\theta$	1	1
6.	Prove $sinh^{-1}x = log_e(x + \sqrt{x^2 + 1})$	1	1

Q. No.	SECTION B ($10 \times 1 = 10$) Answer ALL questions	CO	KL
7.	If $x^3 + px^2 + qx + r = 0$ then the sum of the roots $\alpha + \beta + \gamma =$ _____ $a)\ p$ b) $-p$ c) q d) none	2	2
8.	If a function involving all the roots of an equation is unaltered in value if any two of the roots are interchanged, it is called a _____ a) Symmetric function b) function c) equal function d) none	2	2
9.	The expansion $1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots + \frac{x^n}{n!}$ is equivalent to _____ a) e^{-x} b) e^x c) $e^x + e^{-x}$ d) none	2	2
10.	The expansion $x - \frac{x^2}{2} + \frac{x^3}{3} - \cdots \frac{(-1)^{n-1}x^n}{n} + \cdots$ is equivalent to _____ a) log $(1 - x)$ b) log $(1 + x)$ c) $-$log $(1 + x)$ d) none	2	2
11.	If k is the eigen value of the matrix A, then the eigen value of the matrix A^2 is _____ $a)\ k^2$ b) k c) kA d) none	2	2
12.	If the matrix B is similar to the matrix A, then A and B have the same ____ a) Diagonal b) Rank c) Trace d) Characteristic equation	2	2
13.	$(cos\theta + isin\theta)^n =$ _____ $a)\ cos\theta + isin\theta$ b) $cos\theta + isin\theta$ c) $cosn\theta + isinn\theta$ d) $cosn\theta - isinn\theta$	2	2
14.	$\lim_{\theta \to 0} \frac{sin\theta}{\theta} =$ _____ a) 0 b)1 c) -1 d) none	2	2

15.	$cos\theta + isin\theta =$ _____ $a)\ e^{i\theta}$ b) $e^{-i\theta}$ c) $e^{i\theta} + e^{-i\theta}$ d) none	2	2
16.	$cosh^2x + sinh^2x =$ _____ $a)$ 1 b) $cosh2x$ c) 0 d) none	2	2

Q. No.	SECTION C ($2 \times 15 = 30$) Answer ANY TWO questions	CO	KL
17.	Solve the equation $x^4 - 2x^3 + 4x^2 + 6x - 21 = 0$ given that two of its roots are equal in magnitude and opposite in sign.	3	3
18.	Sum the series $\frac{5}{1!} + \frac{7}{3!} + \frac{9}{5!} + \cdots$	3	3
19.	Examine the eigen values and eigen vectors of the matrix $\begin{bmatrix} 7 & 0 & -2 \\ 0 & 5 & -2 \\ -2 & -2 & 6 \end{bmatrix}$	3	3
20.	Show that $2^5 cos^6\theta = cos6\theta + 6\,cos4\theta + 15\,cos2\theta + 10$.	3	3

Q. No.	SECTION D ($2 \times 15 = 30$) Answer ANY TWO questions	CO	KL
21.	(i) Solve the equation $x^4 - 12x^3 + 48x^2 - 72x + 35 = 0$ by diminishing the roots h so that the second term is absent. (ii) If α, β, γ are the roots of the equation $x^3 + px^2 + qx + r = 0$ find the value of $\sum \alpha^2, \sum \alpha^2\beta$. (10+5)	4	4
22.	(i) Prove that $log\frac{n+1}{n-1} = \frac{2n}{n^2+1} + \frac{1}{3}(\frac{2n}{n^2+1})^3 + \frac{1}{5}(\frac{2n}{n^2+1})^5 + \cdots + \infty$ (ii) Show that $\log_e(1+\frac{1}{n})^n = 1 - \frac{1}{2(n+1)} - \frac{1}{2.3\,(n+1)^2} - \frac{1}{3.4(n+1)^3} - \cdots \infty$ (7+8)	4	4
23.	Diagonalize the matrix $\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$.	4	4
24.	a) If $\sin(A + iB) = x + iy$ prove that (i) $\frac{x^2}{cosh^2B} + \frac{y^2}{sinh^2B} = 1$ (ii) $\frac{x^2}{sin^2A} - \frac{y^2}{cos^2A} = 1$ b) Separate into real and imaginary parts $\tan(x + iy)$ (10+5)	4	4

Q. No.	SECTION E ($2 \times 10 = 20$) Answer ANY TWO questions	CO	KL
25.	Increase the roots of $x^4 - x^3 - 10x^2 + 4x + 24 = 0$ by 2 and hence solve the original equation.	5	5
26.	Evaluate $\lim_{n\to\infty}(1 + \frac{3}{n^2} + \frac{5}{n^3})^{n^2+7n}$	5	5
27.	Analyse Cayley – Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ -2 & 1 & 3 \\ 3 & 2 & -3 \end{bmatrix}$	5	5
28.	Evaluate $cos8\theta$ in terms of $\sin\theta$.	5	5