STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019 – 20 & thereafter)

B. Sc. DEGREE EXAMINATION, NOVEMBER 2023
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE	**:**	**MAJOR – CORE**
PAPER	**:**	**VECTOR ANALYSIS AND APPLICATION**
SUBJECT CODE	**:**	**19MT/MC/VA53**
TIME	**:**	**3 HOURS** **MAX. MARKS : 100**

SECTION – A **(10×2=20)**
ANSWER ANY TEN QUESTIONS

1. Prove that $div\,\vec{r}$=3.
2. State the partial derivative of *r* with respect to *x*.
3. Define surface integral.
4. Prove that $curl\,\vec{r} = 0$.
5. When is a vector function considered to be differentiable?
6. Calculate the work done by the force $\vec{F} = 2\text{y}\ \vec{\imath} + \text{xy}\ \vec{\jmath}$ in moving an object along a straight line from $A(0,0,0)$ to $B(2,1,0)$.
7. State Green's theorem.
8. Find a unit tangent vector to any point on the curve $x = a\cos wt\,, y = a\sin wt$, $z = bt$, where w is a constant.
9. State the physical significance of curl.
10. Define Flux.
11. State the potential energy at any point *P*.
12. Find the unit normal to the surface $x^4 - 3xyz + z^2 + 1 = 0$ at the point $(1, -1, 1)$.

SECTION – B **(5×8=40)**
ANSWER ANY FIVE QUESTIONS

13. If $\vec{a} = \sin\theta\,\vec{\imath} + \cos\theta\,\vec{\jmath} + \theta\vec{k}$, $\vec{b} = \cos\theta\,\vec{\imath} - \sin\theta\,\vec{\jmath} - 3\vec{k}$ and $\vec{c} = 2\ \vec{\imath} + 3\ \vec{\jmath} - 3\vec{k}$, find $\frac{d}{d\theta}\left\{\vec{a}\ \times\left(\vec{b}\ \times\vec{c}\right)\right\}$ at $\theta = \frac{\pi}{2}$.
14. If $\vec{A} = x^2yz\vec{\imath} - 2xy^3\vec{\jmath} + xz^2\vec{k}$ and $\vec{B} = 2z\vec{\imath} + y\vec{\jmath} - x^2\vec{z}$, find $\frac{\partial^2}{\partial x\partial y}\left(\vec{A}\ \times\vec{B}\right)$ at $(1,0,-2)$.
15. Show that $\nabla^2\left(\frac{x}{\vec{r}^3}\right) = 0$.
16. Find a unit vector which is normal to the surface $z = x^2 + y^2$ at the point $(1,2,5)$.
17. Verify Green's theorem in the plane $\oint(xy + y^2)dx + x^2dy$ for a closed region C bounded by $y = x$ and $y = x^2$.
18. Explain in detail the significance of physical interpretation of divergence.
19. Find the total work done in moving a particle in a force field by $\vec{F} = 3\text{xy}\ \vec{\imath} - 5\,\text{z}\ \vec{\jmath} + 10x\vec{k}$ along the curves $x = t^2 + 1, y = 2t^2, z = t^3$ from $t = 1$ to $t = 2$.

SECTION – C **(2×20=40)**

ANSWER ANY TWO QUESTIONS

20. a) If A and B are differentiable vector functions of a scalar t, then prove that

i) $\frac{d}{dt}(A \cdot B) = A \cdot \frac{dB}{dt} + \frac{dA}{dt} \cdot B$ ii) $\frac{d}{dt}(A \times B) = A \times \frac{dB}{dt} + \frac{dA}{dt} \times B$.

b) Find the direction derivative of $\phi = x^2 - y^2 + 2z^2$ at the point $P(1,2,3)$ in the direction of the line PQ, where Q has coordinates $(5,0,4)$.

c) Show that i) $\nabla\left(\frac{1}{r^2}\right) = -\frac{\vec{r}}{r^3}$ ii) $\nabla r^n = nr^{n-2}\vec{r}$ (6+6+8)

21. a) Verify Stoke's theorem for $\vec{F} = y\ \vec{\imath} + z\ \vec{\jmath} + x\vec{k}$, where S is the upper half surface of the sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

b) Calculate the divergence in terms of Curvilinear Coordinates. (10+10)

22. Verify divergence theorem for $\vec{F} = (x^2 - yz)\ \vec{\imath} + (y^2 - zx)\ \vec{\jmath} + (z^2 - xy)\vec{k}$ taken over the rectangular parallelopiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c$.