STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019 – 20 & thereafter)

B. Sc. DEGREE EXAMINATION, NOVEMBER 2023
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE	**:**	**MAJOR – CORE**
PAPER	**:**	**PRINCIPLES OF REAL ANALYSIS**
SUBJECT CODE	**:**	**19MT/MC/RA55**
TIME	**:**	**3 HOURS**

MAX. MARKS : 100

SECTION – A **(10 × 2 = 20)**
ANSWER ANY TEN QUESTIONS

1. If $|x-2|<1$, prove that $|x^2-4|<5$.
2. What is the condition for reformulation in the definition of continuous function?
3. Give an example for metric space.
4. Is $\varphi(t)=\frac{1}{t}$, continuous in $\mathbb{R}$?
5. Define connected subset of a metric space.
6. Is $f(x)=x^3, (0\le x\le 1)$ uniformly continuous?
7. Define compact set of a metric space.
8. When can you say a function *f* is bounded?
9. Define totally bounded subset of a metric space.
10. Define open set in a metric space M.
11. If a real valued function f has a derivative at the point $c\in R^1$, the prove that *f* is continuous at *c*.
12. Is $\int_1^\infty \frac{1}{x^{2/3}}\,dx$ convergent?

SECTION – B **(5 × 8 = 40)**
ANSWER ANY FIVE QUESTIONS

13. If $\lim\limits_{x\to a} f(x)=L$ and $\lim\limits_{x\to a} g(x)=M$, then $f(x)+g(x)$ has a limit as $x\to a$, then prove that $\lim\limits_{x\to a}[f(x)+g(x)]=L+M$.
14. If *f* and g are real valued functions which are continuous at $a\in M$, the prove that $f+g, f-g$ and fg are also continuous at a.
15. Show that a subset *A* of R^1 is connected if and only if whenever $a\in A$, $b\in A$, with $a<b$, then $c\in A$ for any *c* such that $a<c<b$.
16. Prove that if the subset *A* of a metric space $<M,p>$ is totally bounded, then *A* is bounded.

17. Show that the metric space $< M, p >$ is compact if and only if every sequence of points in M has a subsequence converging to a point in M.

18. Let f be a bounded function on the closed bounded interval $[a, b]$. Then prove that $f \in \Re[a, b]$ if and only if for each $\in > 0$, there exists a subdivision σ of $[a, b]$ such that $U[f; \sigma] < L[f; \sigma] + \epsilon$.

19. State and prove Rolle's theorem.

SECTION – C **(2 × 20 = 40)**

ANSWER ANY TWO QUESTIONS

20. a) Let $< M, p >$ be metric space and let a be a point in M. Let f and g be real valued functions whose domains are subsets of M. If $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = N$, then prove that

    i) $\lim_{x \to a} [f(x) + g(x)] = L + N$

    ii) $\lim_{x \to a} [f(x) - g(x)] = L - N$

    iii) $\lim_{x \to a} [f(x) g(x)] = LN$

    iv) $\lim_{x \to a} \frac{f(x)}{g(x)} = \frac{L}{N}, N \neq 0.$

    b) Let E be a subset of the metric space M. Then prove that the point $x \in M$ is a limit point of E if and only if every open ball $B[x; r]$ about x contains at least one point of E. (10+10)

21. a) Let $< M, p >$ be metric space. Then prove that a subset A of M is totally bounded if and only if every sequence of points of A contains a Cauchy subsequence.

    b) If M is a compact metric space then prove that M has the Heine-Borel property. (10+10)

22. a) Let $< M_1, p_1 >$ be a compact metric space. If f is a continuous function from M_1 into a metric space $< M_2, p_2 >$, then prove that f is uniformly continuous on M_1.

    b) If $f \in \Re[a, b]$ and $a < c < b$, then prove that $f \in \Re[a, c]$, $f \in \Re[c, b]$ and $\int_a^b f = \int_a^c f + \int_c^b f$. (10+10)

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