

STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019 – 20 & thereafter)

B. Sc. DEGREE EXAMINATION, NOVEMBER 2023
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE : MAJOR – CORE
PAPER : INTEGRAL TRANSFORMS
SUBJECT CODE : 19MT/MC/IT54
TIME : 3 HOURS

MAX. MARKS : 100

SECTION – A
ANSWER ANY TEN QUESTIONS

(10×2=20)

1. Define piecewise continuity.
2. Find $L(t^2 + 3t + 2023)$.
3. Write the sufficient conditions for the existence of the Laplace transformation.
4. Find $L^{-1} \left[\frac{s}{s^2+a^2} \right]$.
5. State convolution of sequences.
6. Write the complex form of Fourier integrals.
7. What is the inverse Z –transform of $\frac{1}{z-1}$?
8. Define z transform.
9. Find the z transform of $e^{-3t} t$.
10. Define Fourier sine integral.
11. Prove that if $F\{f(x)\} = F(s)$ then $F\{e^{iax} f(x)\} = F(s + a)$.
12. Find the Z- transform of $\frac{1}{n!}$.

SECTION – B
ANSWER ANY FIVE QUESTIONS

(5×8=40)

13. Find $L^{-1} \left[\frac{1}{(s+1)(s^2+2s+2)} \right]$.
14. Solve $\frac{d^2y}{dt^2} + 6 \frac{dy}{dt} + 5y = e^{-2t}$ given that $y(0) = 0, y'(0) = 1$.
15. Determine y which satisfies the equation
 $\frac{dy}{dt} + 3y + 2 \int_0^t y dt = t$ for which $y(0) = 0$.
16. Express the function $f(x) = \begin{cases} 1 & \text{for } |x| \leq 1 \\ 0 & \text{for } |x| > 1 \end{cases}$ as a Fourier integral. Hence evaluate
 $\int_0^\infty \frac{\sin \lambda \cos \lambda}{\lambda} d\lambda$ and $\int_0^\infty \frac{\sin \lambda}{\lambda} d\lambda$.
17. State and prove the change of scale property for Fourier transforms.
18. Obtain the z- transform of $\frac{1}{n+1}$.
19. Using partial fraction method, find the inverse z-transform of $\frac{z^2+z}{(z-1)(z^2+1)}$.

SECTION – C
ANSWER ANY TWO QUESTIONS

(2×20=40)

20. a. Using Laplace transformation solve the system of equations.

$$3 \frac{dx}{dt} + \frac{dy}{dt} = -2x + 1$$

$$\frac{dx}{dt} + 4 \frac{dy}{dt} = -3y.$$

Given that $x(0) = 0, y(0) = 0$.

- b. Find the Fourier cosine transform of $\frac{1}{x^2+a^2}$.

21. a. Evaluate $\int_0^{\infty} \frac{e^{-t}-e^{-2t}}{t} dt$.

- b. State and prove the final value theorem for z -transforms.

22. a. If $F(z) = \frac{2z^2+3z+12}{(z-1)^4}$, find the value of $f(2)$ and $f(3)$.

- b. Solve the difference equation

$$y_{n+2} - 3y_{n+1} - 10y_n = 0, \text{ given that } y_0 = 1, y_1 = 0.$$

