STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted during the academic year 2019–20 and thereafter)

B. Sc. DEGREE EXAMINATION, NOVEMBER 2023
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE   : **MAJOR – CORE**
PAPER   : **ELEMENTS OF GRAPH THEORY**
SUBJECT CODE : **19MT/MC/EG34**
TIME   : **3 HOURS**   **MAX. MARKS: 100**

SECTION-A
Answer any TEN questions   **(10 x 2 = 20)**

1. Define degree of a vertex.

2. Define a complete graph.

3. When do you say a graph is connected?

4. Is the sequence $(4,4,4,2,2,2)$ graphic?

5. What is an Euler trail?

6. Define closure of a graph.

7. Define eccentricity of a tree.

8. When do you say a graph is planar?

9. Prove that K_5 is non – planar .

10. Give an example of a graph which is both Eulerian and Hamiltonian.

11. When do you say a vertex v is reachable?

12. Define source and sink in a directed graph.

SECTION-B
Answer any FIVE questions   **(5 x 8 = 40)**

13. Prove that number of vertices of odd degree is even.

14. Show that any self complementary graph has $4n$ or $4n + 1$ points.

15. Show that closed walk of odd length contains a cycle.

16. Prove that if G is a graph in which degree of every vertex is 2 , then G contains a cycle.

17. Prove that closure of a graph is well defined.

18. Prove that every tree has a centre consisting either of one point or two adjacent points.

19. Prove that if a finite directed graph is cycle – free then prove that it contains a source and sink.

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SECTION-C
Answer any TWO questions   **(2 x 20 = 40)**

20. a) Prove that a line x of a connected graph G is a bridge if and only if x is not on any cycle of G.

    b) Prove that a graph is bipartite if and only if all its cycles are of even length.

                                                        (10+10)

21. a) If G is a graph with $p \geq 3$ vertices and $\delta \geq {}^{p}/_{2}$. Prove that G is Hamiltonian.

    b) Let G be a (p, q) graph. Prove that the following statements are equivalent.

      i) G is a tree.

      ii) Any two points of G is joined by a unique path.

      iii) G is connected and $p = q + 1$.

      iv) G is acyclic and $p = q + 1$.   (10+10)

22. a) State and prove Euler's formula.

    b) Explain Warshal's algorithm.

                                                       (12+8)

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