STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086
(For candidates admitted during the academic year 2019–20 & thereafter)

B. Sc. DEGREE EXAMINATION, NOVEMBER 2023
BRANCH I - MATHEMATICS
FIFTH SEMESTER

COURSE	**:**	**MAJOR – CORE**
PAPER	**:**	**ALGEBRAIC STRUCTURES**
SUBJECT CODE	**:**	**19MT/MC/AS55**
TIME	**:**	**3 HOURS**

MAX. MARKS : 100

SECTION – A

Answer any ten questions: **(10 × 2 =20)**

1. Define a subgroup
2. List the subgroup of Z_{30}.
3. Find the cycles of the permutation
$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 1 & 2 \end{pmatrix}$$
4. Give an example of an automorphism of a group.
5. Define group isomorphism.
6. Define a normal subgroup of a group.
7. Define right coset of H in G.
8. Let $G = S_3$ and $H = \{(1),(13)\}$. Find the left coset of H in G.
9. Give an example of a ring.
10. Define field.
11. Write any two properties of ring homomorphism.
12. Give an example of a maximal ideal.

SECTION – B

Answer any five questions: **(5 × 8 = 40)**

13. Let G be an abelian group and H and K be subgroups of G, then prove that HK is a subgroup of G.
14. Prove that every permutation is a product of 2-cycles.
15. State and prove Fermat's little theorem.
16. Prove that $|HK| = \dfrac{|H||K|}{|H \cap K|}$.
17. Construct multiplication table for $Z_3[i]^*$ and compute it as a ring.
18. Let R be a commutative ring with unity and A be an ideal of R. Then prove that R/A is a field if and only if A is maximal.
19. If F is a field of characteristic p, then F contains a subfield isomorphic to Z_p. If F is a field of characteristic 0, then prove that F contains a subfield isomorphic to the rational numbers.

SECTION – C

Answer any two questions: **($2 \times 20 = 40$)**

20. (a) Let G be a group and let $a \in G$, then prove that $\langle a \rangle$ is a subgroup of G.

    (b) State and prove the Cayley's theorem.

21. (a) Is (1, 2, 3) (1, 2) an even permutation? Why?

    (b) State and prove the Lagrange's theorem.

22. (a) Let D be an integral domain. Then prove that there exist a field F of quotients of D that contains a subring isomorphic to D.

    (b) Prove that a finite integral domain is a field.