

STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI - 600 086
(For candidates admitted during the academic year 2019-2020 and thereafter)

B. Sc. DEGREE EXAMINATION, NOVEMBER 2023
BRANCH I - MATHEMATICS
THIRD SEMESTER

COURSE : ALLIED – CORE
PAPER : MATHEMATICAL STATISTICS – I
SUBJECT CODE : 19MT/AC/ST35
TIME : 3 HOURS **MAX. MARKS : 100**

SECTION – A **(10X2=20)**
ANSWER ANY TEN QUESTIONS

1. Differentiate discrete and continuous random variables.
2. Define cumulative distribution function.
3. What is mathematical expectation?
4. Define moment generating function.
5. Describe a situation that can be transformed into binomial distribution.
6. Under what conditions, Poisson distribution is the limiting form of the binomial distribution?
7. Draw the normal probability curve.
8. State the additive property of the normal distribution.
9. What is a scatter diagram?
10. Write any two demerits of rank correlation coefficient.
11. Find the mean of the random variable X , if its probability density function is given by $f(x) = 6x(1 - x)$ in $0 \leq x \leq 1$.
12. If X is a Poisson random variable such that $E(X^2) = 6$, then find $E(X)$.

SECTION – B **(5X8=40)**
ANSWER ANY FIVE QUESTIONS

13. The joint probability density function of two-dimensional random variables is given by $f(x, y) = x^2y + \frac{y^2}{8}$, $0 \leq x \leq 1$, $0 \leq y \leq 2$. Compute (i) $P(Y > 1)$, (ii) $P(X < \frac{1}{2})$.
14. Prove that $E(X + Y) = E(X) + E(Y)$ and if X and Y are independent random variables, then $E(XY) = E(X)E(Y)$.
15. Derive the recurrence relation for the central moments of the binomial distribution.
16. Find the median and mode of the normal distribution.
17. The ranks obtained by 10 students in mathematics (M) and science (S) are given below.
Find the rank correlation coefficient.

M	6	4	3	1	2	7	9	8	10	5
S	4	1	6	7	5	8	10	9	3	2

18. The joint probability distribution of two discrete random variables is given below.

X	Y		
	1	3	9
2	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$
4	$\frac{1}{4}$	$\frac{1}{4}$	0
6	$\frac{1}{8}$	$\frac{1}{24}$	$\frac{1}{12}$

Find the means of X and Y, and the covariance of X and Y.

19. Explain the probable error in obtaining correlation coefficient.

SECTION – C
ANSWER ANY TWO QUESTIONS

(2X20=40)

20. Prove the Baye's theorem for future events and apply to solve the following: Suppose that a product is produced in three factories X, Y, and Z. It is known that factory X produces thrice as many times as factory Y, and that factories Y and Z produce the same number of items. Assume that it is known that 3 percent of the items produced by each of the factories X and Z are defective while 5 percent of those manufactured by factory Y are defective. All the items produced in the three factories are stocked, and an item of product is selected at random. (i) What is the probability that this item is defective? (ii) If an item selected at random is found to be defective, what is the probability that it was produced by factory X, Y, and Z respectively?

21. (a) Find the characteristic function of the distribution $P(X = r) = pq^r, r = 0, 1, 2, \dots$ where $p + q = 1$. Hence find the mean and variance of the distribution. (10)

(b) Obtain the mode of the Poisson distribution. (10)

22. (a) Obtain the moment generating function of normal distribution. (10)

(b) Compute the correlation coefficient for the following data. (10)

X	67	71	70	66	68	67	69	65
Y	64	70	67	68	72	68	70	67

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