**STELLA MARIS COLLEGE (AUTONOMOUS) CHENNAI 600 086**
**(For candidates admitted during the year 2019-20 and thereafter)**

**SUBJECT CODE: 19MT/PE/MS15**

**M. Sc. DEGREE EXAMINATION, APRIL 2023**
**BRANCH I - MATHEMATICS**
**FOURTH SEMESTER**

**COURSE : ELECTIVE**
**PAPER : MATHEMATICAL STATISTICS**
**TIME : 3 HOURS** **MAX. MARKS : 100**

**SECTION – A**
**ANSWER ALL THE QUESTIONS (5 × 2 = 10)**

1. Write a necessary and sufficient condition for a function $\varphi(t)$ to be a characteristic function of some distribution function.
2. Can you say the variance of a random variable equals zero, when $x$ follows one point distribution? Justify.
3. When do you say the distribution function $F(x)$ is the limit distribution function?
4. Define Fisher's Z-distribution.
5. Define Unbiased Estimate.

**SECTION – B**
**ANSWER ANY FIVE QUESTIONS (5 × 6 = 30)**

6. Obtain the Characteristic function of a normal distribution and hence find the first two moments $m_1$ and $m_2$.
7. Find the mean and variance of a Gamma distribution.
8. State and prove Bernoulli's law of large numbers.
9. Define student's $t$-distribution with $n$ degrees of freedom and derive its density function.
10. Find the MLE for the parameter $\lambda$ of a Poisson distribution on the basis of sample size $n$.
11. State and prove any four properties of Characteristic function.
12. State and prove Laplace de Moivre's theorem.

**SECTION – C**
**ANSWER ANY THREE QUESTIONS (3 × 20 = 60)**

13. Prove that $F(a+h) - F(a-h) = \lim_{T\to\infty} \frac{1}{\pi}\int_{-T}^{T} \frac{\sin ht}{t} e^{-tia}\varphi(t)dt$, where $F(x)$ and $\varphi(t)$ denote respectively the distribution function and the characteristic function of the random variable $X$ and $a+h$ and $a-h$ are the continuity points of $F(x)$. Hence find the density function of the random variable $X$, whose characteristic function is $\varphi(t) = \begin{cases} 1-|t| & for\ |t| \le 1 \\ 0 & for\ |t| > 1 \end{cases}$.

14. a) Obtain the $k^{th}$ moment of Beta distribution and hence find mean and variance.

    b) Prove that the Characteristic function of a Cauchy distribution is $e^{-|t|}$ .

15. State and Prove Levy-Cramer theorem.

16. Discuss Chi-square distribution function and obtain Moment generating function hence find mean and variance.

17. State and prove Rao-Cramer Inequality.

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